UNIT 1 SIMPLE MECHANISMS

Structure

1.1 INTRODUCTION

In our daily life, we come across a wide array of machines. It can be a sewing machine, a cycle or a motor car. Power is produced by the engine which makes use of a mechanism called slider crank mechanism. It converts reciprocating motion of a piston into rotary motion of the crank. The power of the engine is transmitted to the wheels with the help of different mechanisms. If you visit LPG gas filling plant or a bottling plant almost all the functions are done by making use of mechanisms. These are only few examples. Generally, manual handling in industries has been reduced to the minimum. In engineering, mechanisms and machines are two very common and frequently used terms. We shall start with simple definition of these terms.

In this unit, you will also study about link, mechanism, machine, kinematic quantities, different types of motion and planar mechanism. You will study about degree of freedom, kinematic pairs and classification of links in this unit.

A moving body has to be assigned coordinates according to the axes assigned. The motion of the bodies is constrained according to the requirement in a mechanism. The links which are the basic elements of the mechanism are connected to form kinematic pairs which are of different types. The links may further be connected to several links in order to impart motion and they are classified accordingly.

In this unit, you will be explained how to get different mechanisms by using four bar chain which is a basic kinematic chain. The four bar chain has four links which are connected with each other with the help of four lower kinematic pairs. This chain provides different mechanisms of common usage. For example, one mechanism, provided by this, is used in petrol engine, diesel engine, steam engine, compressors, etc. One mechanism makes possible to complete idle stroke in machine tool in lesser time than cutting stroke which reduces machining time. This mechanism being termed as quick return mechanism. Similarly, there are some mechanisms which can provide rocking motion which can be used in materials handling. You will be explained terminology and classification of cams and followers also.

Objectives

After studying this unit, you should be able to

- determine degrees of freedom for a link and kinematic pair,
- describe kinematic pair and determine motion,
- distinguish and categorise different type of links,
- know inversions of different kinematic chains,
- understand utility of various mechanisms of four bar kinematic chain,
- make kinematic design of a mechanism,
- know special purpose mechanisms,
- know terminology of cams, and
- know classification of followers and cams.

1.2 KINEMATICS OF MACHINES

The kinematics of machines deals with analysis and synthesis of mechanisms. Before proceeding to this, you are introduced to the kinematics.

Kinematics implies displacement, velocity and acceleration of a point of interest at a particular time or with passage of time. A point or a particle may be displaced from its initial position in any direction. The motion of a particle confined to move in a plane can be defined by x, y or r, θ or some other pair of independent coordinates. The motion of a particle constrained to move along a straight line can be defined by any one coordinate. The concerned coordinate shall describe its location at any instant.

1.2.1 Displacement

The distance of the position of the point from a fixed reference point is called displacement.

In rectilinear motion the displacement, is along one axis say x-axis, therefore,

Displacement 's' $= x$

In a general plane motion,

Displacement 's' $= x + i\,y$

1.2.2 Velocity

The velocity of a particle is defined as the rate of change of displacement, therefore, the velocity

$$V = \frac{S_2 - S_1}{t_2 - t_1} = \frac{\Delta s}{\Delta t}$$

where, $\Delta s = S_2 - S_1$

and $\Delta t = t_2 - t_1$

Δs is the distance traveled in time Δt. The direction of velocity shall be tangent to the path of motion.

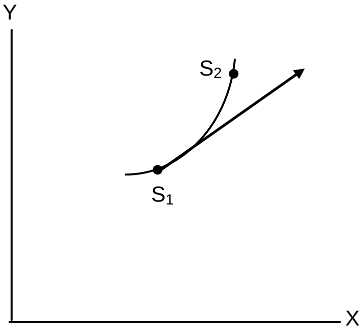


Figure 1.1 : Plane Motion

1.2.3 Acceleration

The acceleration of a particle is defined as the rate of change of velocity, therefore,

$$\text{Acceleration ‘}a\text{’} = \frac{V_2 - V_1}{t_2 - t_1} = \frac{\Delta V}{\Delta t}$$

where $\Delta V = V_2 - V_1$

and $\Delta t = t_2 - t_1$

ΔV is the change in velocity in time Δt.

1.3 KINEMATIC LINK OR AN ELEMENT

Machines consist of several material bodies, each one of them being called link or kinematic link or an element. It is a resistant body or an assembly of resistant bodies. The deformation, if any, due to application of forces is negligible. If a link is made of several resistant bodies, they should form one unit with no relative motion of parts with respect to each other.

For example, piston, piston rod and cross head in steam engine consist of different parts but after joining together they do not have relative motion with respect to each other and they form one link. Similarly, ropes, belts, fluid in hydraulic press, etc. undergo small amount of deformation which, if neglected, will work as resistant bodies and, thereby, can be called links.

SAQ 1

(a) What is a resistant body?

(b) Define link.

1.4 CLASSIFICATION OF LINKS

A resistant body or group of resistant bodies with rigid connections preventing their relative movement is known as a link.

The links are classified depending on number of joints.

Singular Link

A link which is connected to only one other link is called a singular link (Figure 1.2).

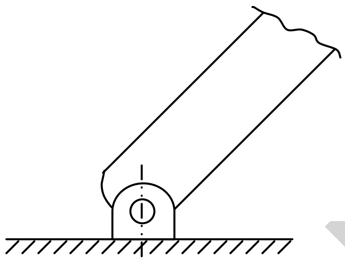


Figure 1.2 : Singular Link

Binary Link

A link which is connected to two other links is called a binary link (Figure 1.3).

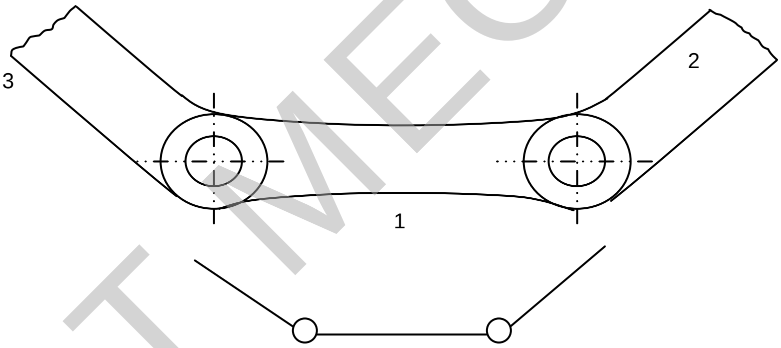


Figure 1.3 : Binary Link

Ternary Link

A link which is connected to three other links is called a ternary link (Figure 1.4).

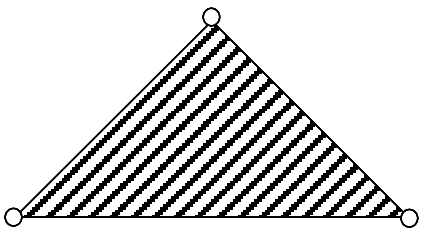


Figure 1.4 : Ternary Link

Quarternary Link

A link which is connected to four other links is called quarternary link (Figure 1.5).

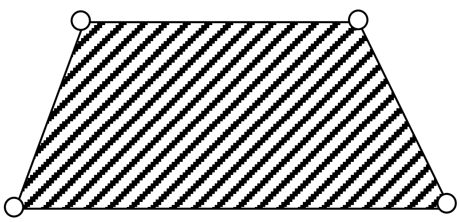


Figure 1.5 : Quarternary Link

1.5 DEGREE OF FREEDOM

The degree of freedom of a body is equal to the number of independent coordinates required to specify the movement. For a cricket ball when it is in air, six independent coordinates are required to define its motion. Three independent displacement coordinates along the three axes (*x*, *y*, *z*) and three independent coordinates for rotations about these axes are required to describe its motion in space. Therefore, degrees of freedom for this ball is equal to six. If this cricket ball moves on the ground, this movement can be described by two axes in the plane.

When the body has a plane surface to slide on a plane, the rotation about *x* and *y*-axes shall be eliminated but it can rotate about an axis perpendicular to the plane, i.e. *z*-axis. At the same time, while executing plane motion, this body undergoes displacement which can be resolved along *x* and *y* axis. The rotation about *z*-axis and components of displacement along *x* and *y* axes are independent of each other. Therefore, a sliding body on a plane surface has three degrees of freedom.

These were the examples of unconstrained or partially constrained bodies. If a cylinder rolls without sliding along a straight guided path, the degree of freedom is equal to one only because rotation in case of pure rolling is dependent on linear motion. This is a case of completely constrained motion.

The angle of rotation $\theta = \frac{x}{r}$

where, *r* is radius of cylinder and *x* is linear displacement.

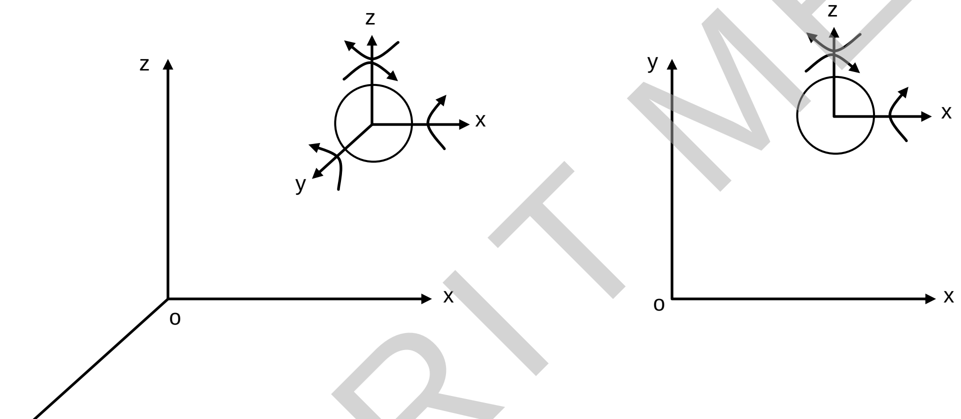


Figure 1.6 : Degree of Freedom

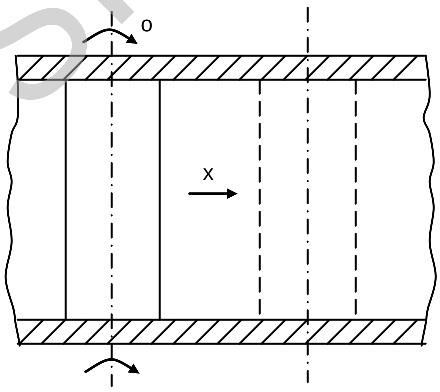


Figure 1.7 : Completely Constrained Motion

1.6 KINEMATIC PAIRS

In a mechanism, bodies or links are connected such that each of them moves with respect to another. The behaviour of the mechanism depends on the nature of the connections of the links and the type of relative motion they permit. These connections are known as kinematic pairs. Hence kinematic pair is defined as a joint of two links having relative motion between them.

Broadly, kinematic pairs can be classified as :

(a) Lower pair,

(b) Higher pair, and

(c) Wrapping pair.

1.7 DIFFERENT PAIRS

When connection between two elements is through the area of contact, i.e. there is surface contact between the two links, the pair is called lower pair. Examples are motion of slider in the cylinder, motion between crank pin and connecting rod at big end.

1.7.1 Types of Lower Pairs

There are six types of lower pairs as given below :

(a) Revolute or Turning Pair (Hinged Joint)

(b) Prismatic of Sliding Pair

(c) Screw Pair

(d) Cylindrical Pair

(e) Spherical Pair

(f) Planar Pair

Revolute or Turning Pair (Hinged Joint)

A revolute pair is shown in Figure 1.8. It is seen that this pair allows only one relative rotation between elements 1 and 2, which can be expressed by a single coordinate ‘θ’. Thus, a revolute pair has a single degree of freedom.

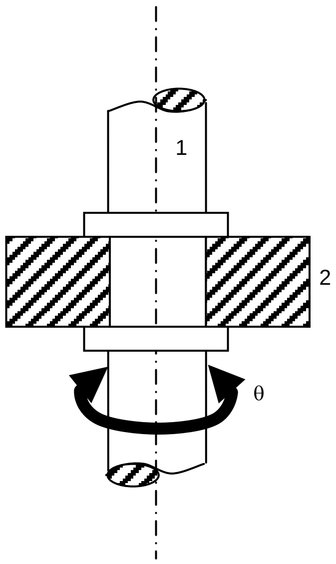


Figure 1.8 : Revolute or Turning Pair

Prismatic or Sliding Pair

As shown in Figure 1.9, a prismatic pair allows only a relative translation between elements 1 and 2, which can be expressed by a single coordinate ‘*s*’, and it has one degree of freedom.

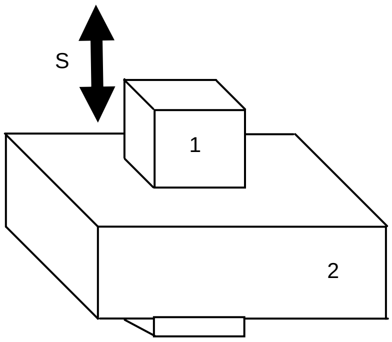


Figure 1.9 : Prismatic or Sliding Pair

Screw Pair

As shown in Figure 1.10, a screw pair allows rotation as well as translation but these two movements are related to each other. Therefore, screw pair has one degree of freedom because the relative movement between 1 and 2 can be expressed by a single coordinate 'θ' or '*s*'. These two coordinates are related by the following relation :

$$\frac{\Delta\theta}{2\pi} = \frac{\Delta s}{L}$$

where, *L* is lead of the screw.

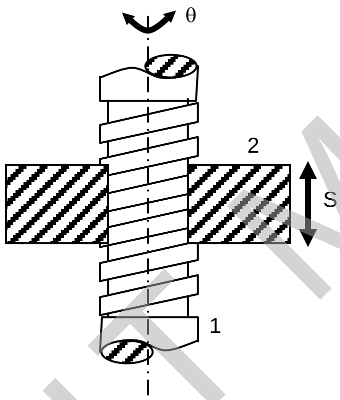


Figure 1.10 : Screw Pair

Cylindrical Pair

As shown in Figure 1.11, a cylindrical pair allows both rotation and translation parallel to the axis of rotation between elements 1 and 2. These relative movements can be expressed by two independent coordinates 'θ' or '*s*' because they are not related with each other. Degrees of freedom in this case are equal to two.

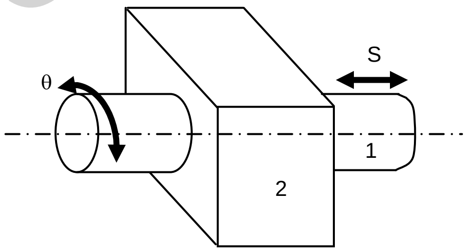


Figure 1.11 : Cylindrical Pair

Spherical Pair

A ball and socket joint, as shown in Figure 1.12, forms a spherical pair. Any rotation of element 2 relative to 1 can be resolved in the three components. Therefore, the complete description of motion requires three independent coordinates. Two of these coordinates 'α' and 'β' are required to specify the position of axis *OA* and the third coordinate 'θ' describes the rotation about the axis of *OA*. This pair has three degrees of freedom.

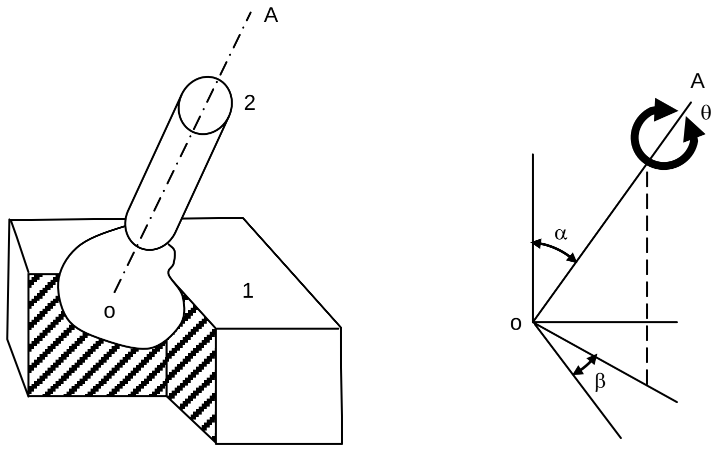


Figure 1.12 : Spherical Pair

Planar Pair

A planar pair is shown in Figure 1.13. The relative motion between 1 and 2 can be described by *x* and *y* coordinates in *x*-*y* plane. The *x* and *y* coordinates describe relative translation and θ describes relative rotation about *z*-axis. This pair has three degrees of freedom.

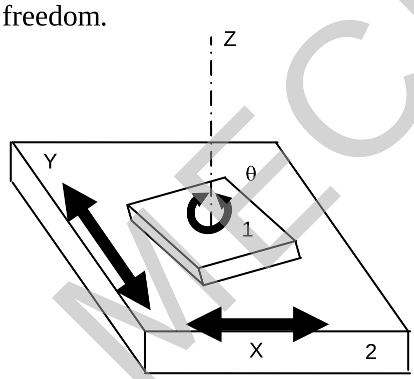


Figure 1.13 : Planar Pair

1.7.2 Higher Pair

A higher pair is a kinematic pair in which connection between two elements is only a point or line contact. The cam and follower arrangement shown in Figure 1.14 is an example of this pair. The contact between cam and flat faced follower is through a line. Other examples are ball bearings, roller bearings, gears, etc. A cylinder rolling on a flat surface has a line contact while a spherical ball moving on a flat surface has a point contact.

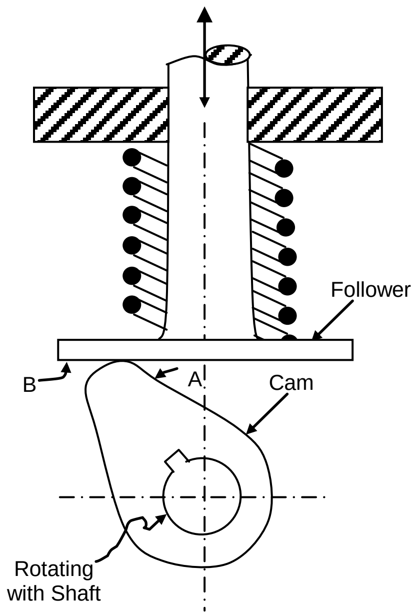


Figure 1.14 : Higher Pair

1.7.3 Wrapping Pair

Wrapping pairs comprise belts, chains and such other devices. Belt comes from one side of the pulley and moves over to other side through another pulley as shown in Figure 1.15.

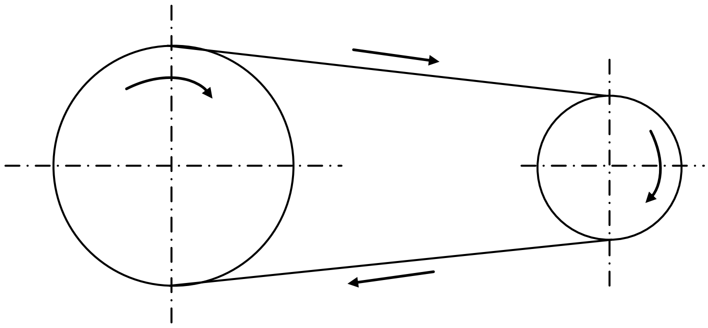


Figure 1.15 : Wrapping Pair

1.8 KINEMATIC CHAINS

In a kinematic chain, four links are required which are connected with each other with the help of lower pairs. These pairs can be revolute pairs or prismatic pairs. A prismatic pair can be thought of as the limiting case of a revolute pair.

Before going into the general theory of mechanisms it may be observed that to form a simple closed chain we need at least three links with three kinematic pairs. If any one of these three links is fixed, there cannot be relative movement and, therefore, it does not form a mechanism but it becomes a structure which is completely rigid. Thus, a simplest mechanism consists of four links, each connected by a kinematic lower pair (revolute etc.), and it is known as four bar mechanism.

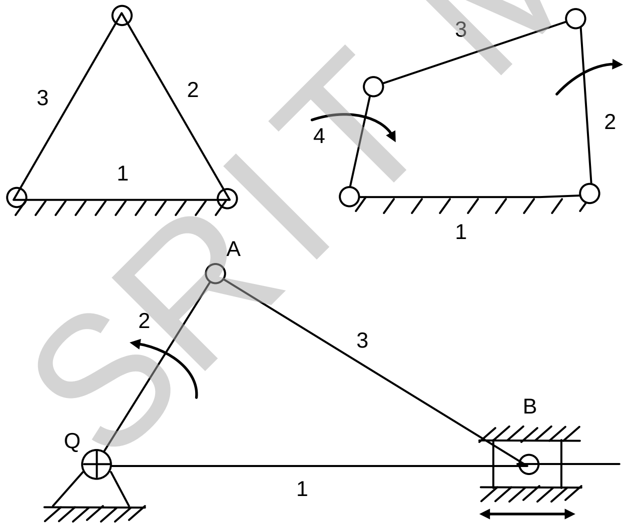


Figure 1.16 : Planar Mechanism

For example, reciprocating engine mechanism is a planner mechanism in which link 1 is fixed, link 2 rotates and link 4 reciprocates. In internal combustion engines, it converts reciprocating motion of piston into rotating motion of crank. This mechanism is also used in reciprocating compressors in which it converts rotating motion of crank into reciprocating motion of piston. This was a very common practical example and there are many other examples like this. More about planar mechanisms shall follow in following sections.

Let us consider the two mechanisms shown in Figure 1.17. The curved slider in figure acts similar to the revolute pair. If radius of curvature ‘ρ’ of the curved slider becomes infinite, the angular motion of the slider changes into linear displacement and the revolute pair R_4 transforms to a prismatic pair. Depending on different type of kinematic pairs, four bar kinematic chain can be classified into three categories :

(a) 4R-kinematic chain which has all the four kinematic pairs as revolute pairs.

(b) 3R-1P kinematic chain which has three revolute pairs and one prismatic pair. This is also called as single slider crank chain.

(c) 2R-2P kinematic chain which has two revolute pairs and two prismatic pairs. This is also called as double slider crank chain.

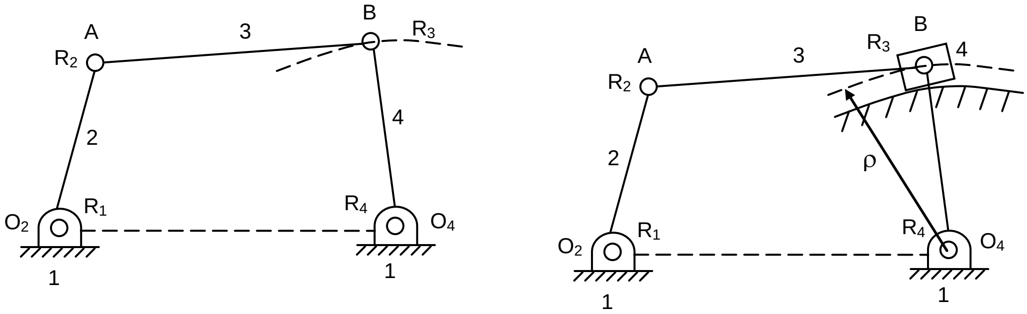


Figure 1.17 : Kinematic Chain

SAQ 2

Form a kinematic chain using three revolute pairs and one prismatic pair.

1.9 INVERSIONS OF KINEMATIC CHAIN

If in a four bar kinematic chain all links are free, motion will be unconstrained. When one link of a kinematic chain is fixed, it works as a mechanism. From a four link kinematic chain, four different mechanisms can be obtained by fixing each of the four links turn by turn. All these mechanisms are called inversions of the parent kinematic chain. By this principle of inversions of a four link chain, several useful mechanisms can be obtained.

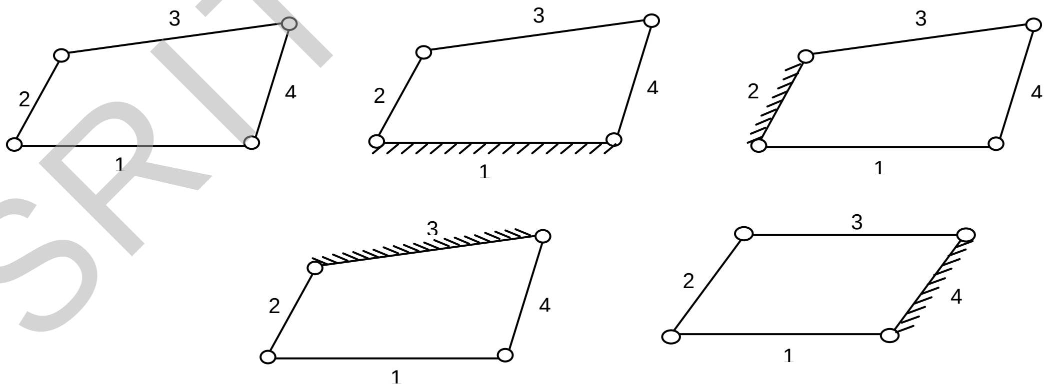


Figure 1.18 : Inversion of Kinematic Chain

1.9.1 Inversions of 4R-Kinematic Chain

Kinematically speaking, all four inversions of 4R-kinematic chain are identical. However, by suitably altering the proportions of lengths of links 1, 2, 3 and 4 respectively several mechanisms are obtained. Three different forms are illustrated here. In Figure 1.19, links have been shown by blocks and lines connecting them represent pairs.

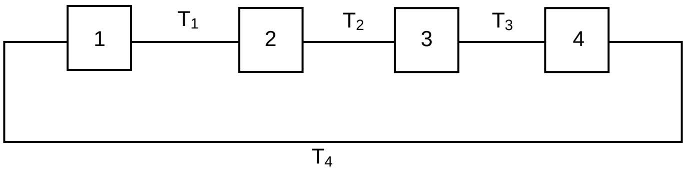


Figure 1.19 : First Inversion

Crank-lever Mechanism or Crank-rocker Mechanism

This mechanism is shown in Figure 1.20. In this case for every complete rotation of link 2 (called a crank), the link 4 (called a lever or rocker), makes oscillation between extreme positions O_4B_1 and O_4B_2.

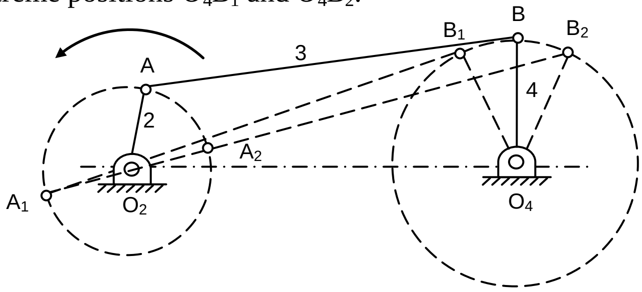


Figure 1.20 : Crank-rocker Mechanism

The position of O_4B_1 is obtained when point A is A_1 whereas position O_4B_2 is obtained when A is at A_2. It may be observed that crank angles for the two strokes (forward and backward) of oscillating link O_4B are not same. It may also be noted that the length of the crank is very short. If l_1, l_2, l_3 and l_4 are lengths of links 1, 2, 3 and 4 respectively, the proportions of the link may be as follows :

$$(l_1 + l_2) < (l_3 + l_4)$$

$$(l_2 + l_3) < (l_1 + l_4)$$

Double-leaver Mechanism or Rocker-Rocker Mechanism

In this mechanism, both the links 2 and 4 can only oscillate. This is shown in Figure 1.21.

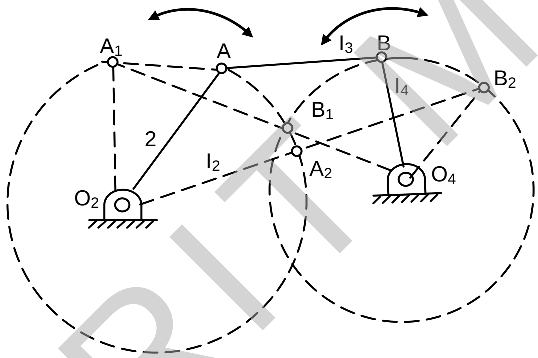


Figure 1.21 : Double-lever Mechanism

Link O_2A oscillates between positions O_2A_1 and O_2A_2 whereas O_4B oscillates between positions O_4B_1 and O_4B_2. Position O_4B_2 is obtained when O_2A and AB are along straight line and position O_2A_1 is obtained when AB and O_4B are along straight line. This mechanism must satisfy the following relations.

$$(l_3 + l_4) < (l_1 + l_2)$$

$$(l_2 + l_3) < (l_1 + l_4)$$

It may be observed that link AB has shorter length as compared to other links.

If links 2 and 4 are of equal lengths and $l_1 > l_3$, this mechanism forms automobile steering gear.

Double Crank Mechanism

The links 2 and 4 of the double crank mechanism make complete revolutions. There are two forms of this mechanism.

Parallel Crank Mechanism

In this mechanism, lengths of links 2 and 4 are equal. Lengths of links 1 and 3 are also equal. It is shown in Figure 1.22.

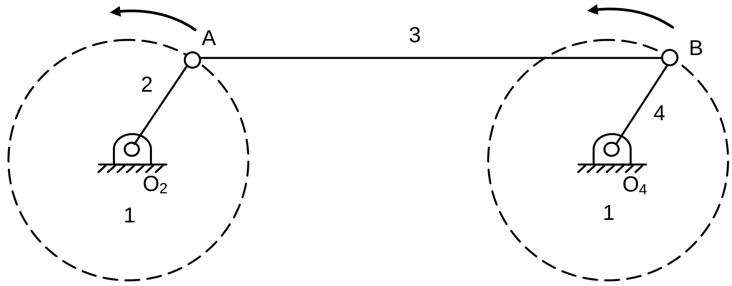


Figure 1.22 : Double Crank Mechanism

The familiar example is coupling of the locomotive wheels where wheels act as cranks of equal length and length of the coupling rod is equal to centre distance between the two coupled wheels.

Drag Link Mechanism

In this mechanism also links 2 and 4 make full rotation. As the link 2 and 4 rotate sometimes link 4 rotate faster and sometimes it becomes slow in rotation.

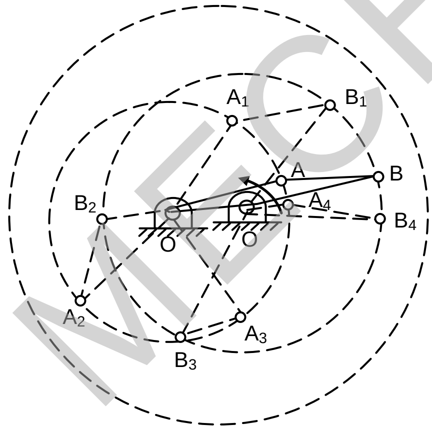


Figure 1.23 : Drag Link Mechanism

The proportions of this mechanism are

$$l_3 > l_1 \;\; ; \;\; l_4 > l_2$$

$$l_3 > (l_1 + l_4 - l_2)$$

and $$l_3 < (l_2 + l_4 - l_1)$$

It may be observed from Figure 1.23 that length of link 1 is smaller as compared to other links.

SAQ 3

Why 4R-kinematic chain does not provide four different mechanisms?

SAQ 4

In this mechanism, if length of link 2 is equal to that of link 4 and link 4 has lengths equal to that of link 2 which mechanism will result and analyse motion.

1.9.2 Inversions of 3R-1P Kinematic Chain or Inversions of Slider Crank Chain

In this four bar kinematic chain, four links shown by blocks are connected through three revolute pairs T_1, T_2 and T_3 and one prismatic pair.

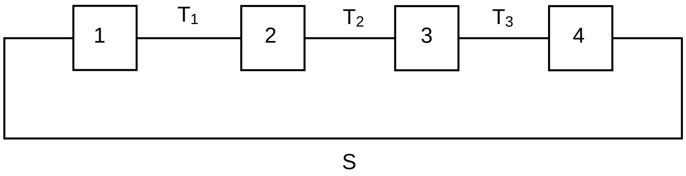


Figure 1.24 ; Inversion of Slider Crank Chain

First Inversion

In this mechanism, link 1 is fixed, link 2 works as crank, link 4 works as a slider and link 3 connects link 2 with 4. It is called connecting rod. Between links 1 and 4 sliding pair has been provided.

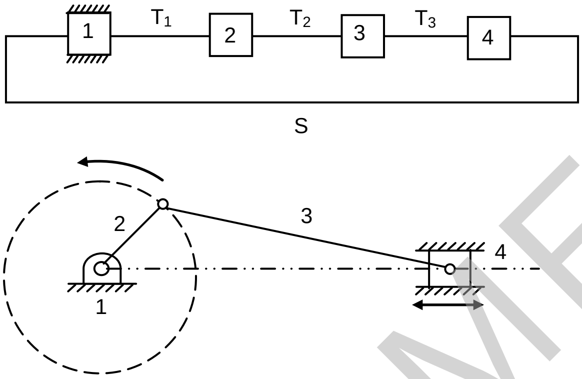


Figure 1.25 : First Inversion

This mechanism is also known as slider crank chain or reciprocating engine mechanism because it is used in internal combustion engines. It is also used in reciprocating pumps as it converts rotatory motion into reciprocating motion and vice-versa.

Second Inversion

In this case link 2 is fixed and link 3 works as crank. Link 1 is a slotted link which facilitates movement of link 4 which is a slider. This arrangement gives quick return motion mechanism. The motion of link 1 can be taped through a link and provided to ram of shaper machine. Figure 1.26 shows this mechanism and it is called Whitworth Quick Return Motion Mechanism. The forward stroke starts when link 3 occupies position O_4Q. At that time, point A is at A_1. The forward stroke ends when link 3 occupies position O_4P and point A occupies position A_2. The return stroke takes place when link 3 moves from position O_4P to O_4Q. The stroke length is distance between A_1 and A_2 along line of stroke. If acute angle < $PO_4\,Q = \theta$ and crank rotates at constant speed ω.

$$\text{Quick Return Ratio} = \frac{\text{Time taken in forward stroke}}{\text{Time taken in return stroke}}$$

$$= \frac{\frac{(2\pi - \theta)}{\omega}}{\frac{\theta}{\omega}}$$

$$= \frac{2\pi - \theta}{\theta}$$

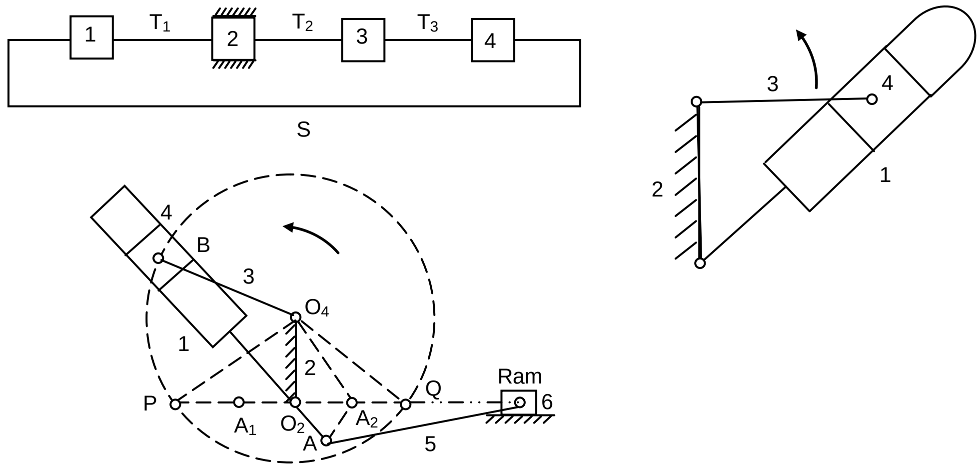
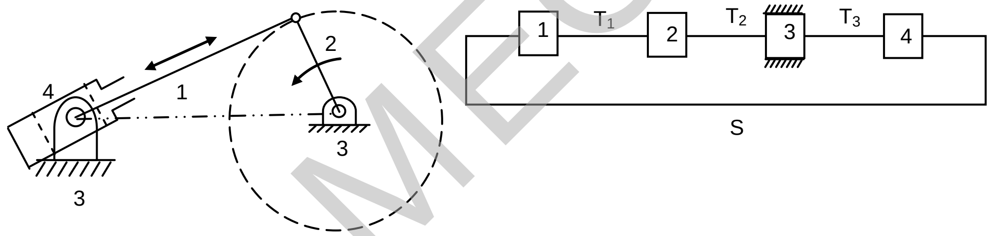


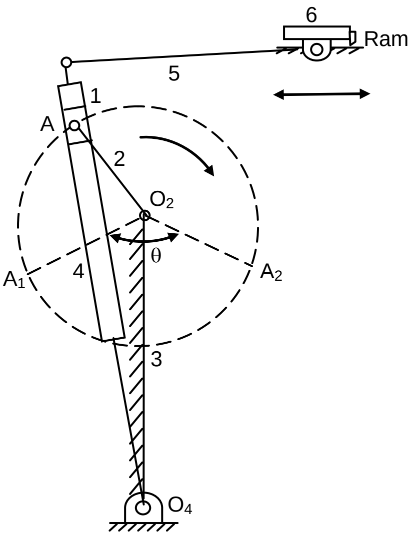
Figure 1.26 : Second Inversion

Third Inversion

This inversion is obtained by fixing link 3. Some applications of this inversion are oscillating cylinder engine and crank and slotted lever quick return motion mechanism of a shaper machine. Link 1 works as a slider which slides in slotted or cylindrical link 4. Link 2 works as a crank. The oscillating cylinder engine is shown in Figure 1.27(a).



(a) Oscillating Cylinder Engine



(b) Crank and Slotted Lever Mechanism

Figure 1.27 : Third Inversion

The motion of link 4 in crank and slotted lever quick return motion mechanism can be taped through link 5 and can be transferred to ram. O_2A_1 and O_2A_2 are two positions of crank when link 4 will be tangential to the crank circle and corresponding to which ram will have extreme positions. When crank travels from position O_2A_1 and O_2A_2 forward stroke takes place. When crank moves further from position O_2A_2 to O_2A_1 return stroke takes place. Therefore, for constant angular velocity for crank 'ω'.

$$\text{Quick Return Ratio} = \frac{\text{Time for forward stroke}}{\text{Time for return stroke}} = \frac{\frac{(2\pi - \theta)}{\omega}}{\frac{\theta}{\omega}} = \frac{2\pi - \theta}{\theta}$$

Fourth Inversion – Pendulum Pump

It is obtained by fixing link 4 which is slider. Application of this inversion is limited. The pendulum pump and hand pump are examples of this inversion. In pendulum pump, link 3 oscillates like a pendulum and link 1 has translatory motion which can be used for a pump.

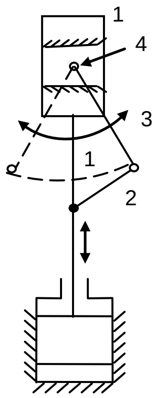


Figure 1.28(a) : Pendulum Pump

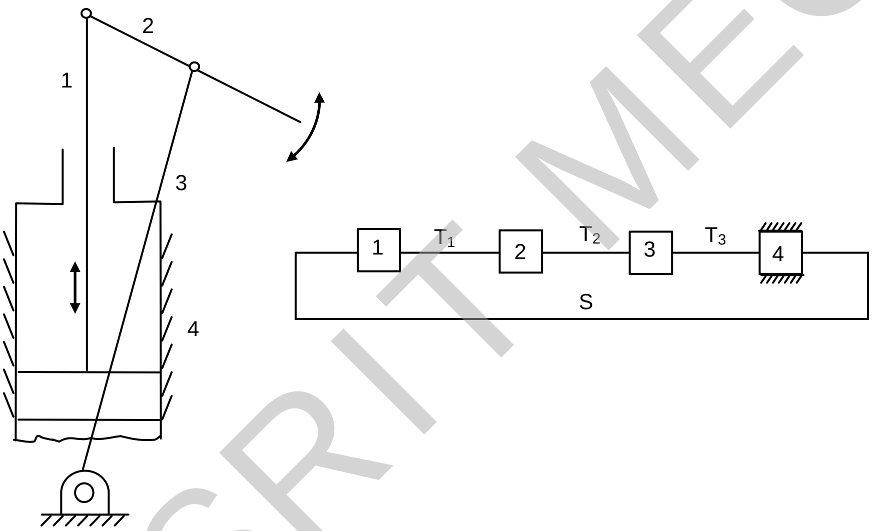


Figure 1.28(b) : Hand Pump

1.9.3 Inversions of 2R-2P Kinematic Chain or Double Slider Crank Chain

This four bar kinematic chain has two revolute or turning pairs – T_1 and T_2 and two prismatic or sliding pairs – S_1 and S_2. This chain provides three different mechanisms.

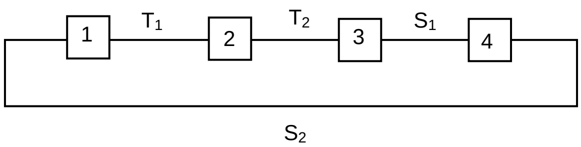


Figure 1.29 : Inversion of 2R-2P Kinematic Chain

First Inversion

The first inversion is obtained by fixing link 1. By doing so a mechanism called Scotch Yoke is obtained. The link 1 is a slider similar to link 3. Link 2 works as a crank. Link 4 is a slotted link. When link 2 rotates, link 4 has simple harmonic motion for angle ‘θ’ of link 2, the displacement of link 4 is given by

$$x = OA \cos \theta$$

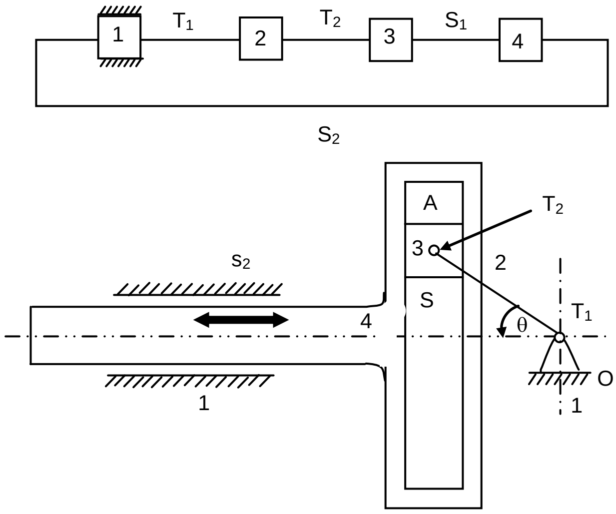


Figure 1.30 : Scotch Yoke Mechanism

Second Inversion

In this case, link 2 is fixed and a mechanism called Oldham's coupling is obtained. This coupling is used to connect two shafts which have eccentricity 'ε'. The axes of the two shafts are parallel but displaced by distance ε. The link 4 slides in the two slots provided in links 3 and 1. The centre of this link will move on a circle with diameter equal to eccentricity.

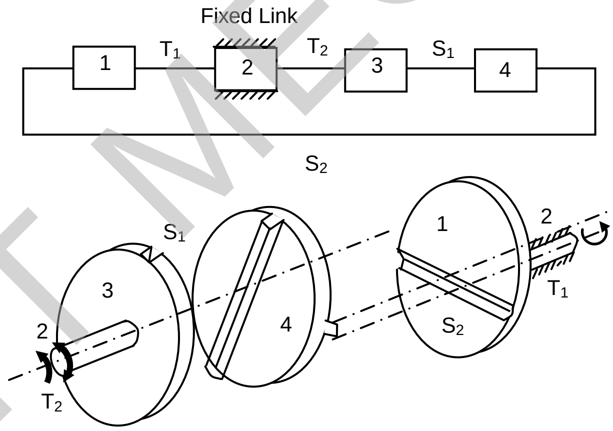


Figure 1.31 : Oldham's Coupling

Third Inversion

This inversion is obtained by fixing link 4. The mechanism so obtained is called elliptical trammel which is shown in Figure 1.32. This mechanism is used to draw ellipse. The link 1, which is slider, moves in a horizontal slot of fixed link 4. The link 3 is also a slider moves in vertical slot. The point *D* on the extended portion of link 2 traces ellipse with the system of axes shown in the figure, the position coordinates of point *D* are as follows :

$$x_D = AD \sin \theta \quad \text{or} \quad \sin \theta = \frac{x_D}{AD}$$

$$y_D = CD \cos \theta \quad \text{or} \quad \cos \theta = \frac{y_D}{CD}$$

Since, $\sin^2 \theta + \cos^2 \theta = 1$

Substituting for sin θ and cos θ in this equation, the following equation of ellipse is obtained.

$$\frac{x_D^2}{AD^2} + \frac{y_D^1}{CD^2} = 1$$

The semi-major axis of the ellipse is *AD* and semi-minor axis is *CD*.

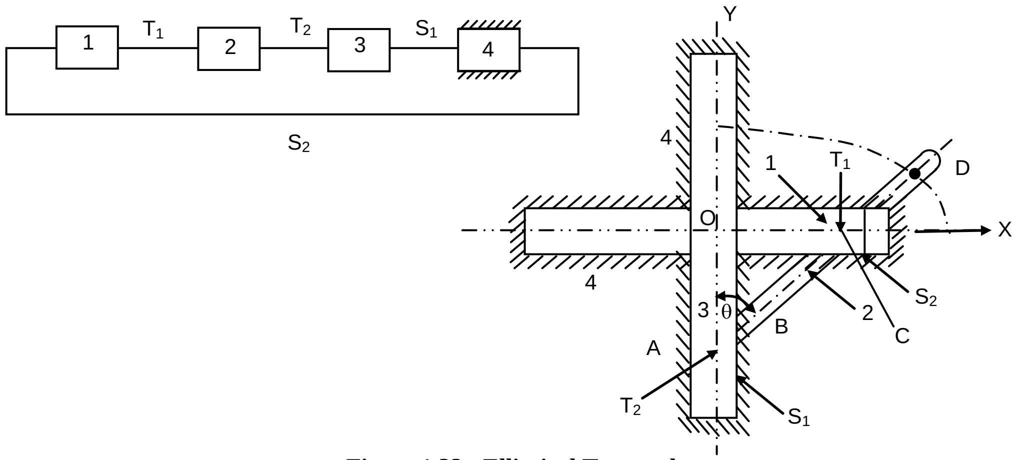


Figure 1.32 : Elliptical Trammel

SAQ 5

(a) Explain why only three different mechanisms are available from 2R-2P kinematic chain.

(b) If length of crank in the reciprocating mechanism is 15 cm, find stroke length of the slider.

(c) If length of fixed link and crank in crank and slotted lever quick return mechanism are 30 cm and 15 cm respectively, determine quick return ratio.

(d) If an ellipse of semi-major axis 30 cm and semi-minor axis 20 cm is to be drawn, what should be the length of link *ACD* in elliptical trammel.

1.10 MACHINE

A machine is a mechanism or collection of several mechanisms which transmits force from power source to the resistance to be overcome and, thereby, it performs useful mechanical work. A common type of example is the commonly used internal-combustion engine. The burning of petrol or diesel in cylinder results in a force on the piston which is transmitted to the crank to result in driving torque. This driving torque overcomes the resistance due to any external agency or friction, etc. at the crankshaft and thereby doing useful work.

1.10.1 Difference between Machine and Mechanism

A system can be defined as a mechanism or a machine on the basis of primary objective.

Sl. No.	Machine	Mechanism
1	If the system is used with the objective of transforming mechanical energy, then it is called a machine	If the objective is to transfer or transform motion without considering forces involved, the system is said to be a mechanism
2	Every machine has to transmit motion because mechanical work is associated with the motion, and thus makes use of mechanisms	It is concerned with transfer of motion only
3	A machine can use one or more than one mechanism to perform the desired function, e.g. sewing machine has several mechanisms	It is not the case with mechanisms. A mechanism is a single system to transfer or transform motion

1.11 OTHER MECHANISMS

Geometry of motion of well known lower pair mechanisms will be examined and their actual working and application will be dealt with. On the strength of analytical study of these mechanisms, new mechanisms can be developed for specific requirements, for modern plants and machinery.

1.11.1 Pantograph

Pantograph is a geometrical instrument used in drawing offices for reproducing given geometrical figures or plane areas of any shape, on an enlarged or reduced scale. It is also used for guiding cutting tools. Its mechanism is utilised as an indicator rig for reproducing the displacement of cross-head of a reciprocating engine which, in effect, gives the position of displacement.

There could be a number of forms of a pantograph. One such form is shown in Figure 1.33. It comprises of four links : *AB*, *BC*, *CD*, *DA*, pin-jointed at *A*, *B*, *C* and *D*. Link *BA* is extended to a fixed pin *O*. Suppose *Q* is a point on the link *AD* of which the motion is to be enlarged, then the link *BC* is extended to *P* such that *O*, *Q*, *P* are in a straight line. It may be pointed out that link *BC* is parallel to link *AD* and that *AB* is parallel to *CD* as shown. Thus, *ABCD* is a parallelogram.

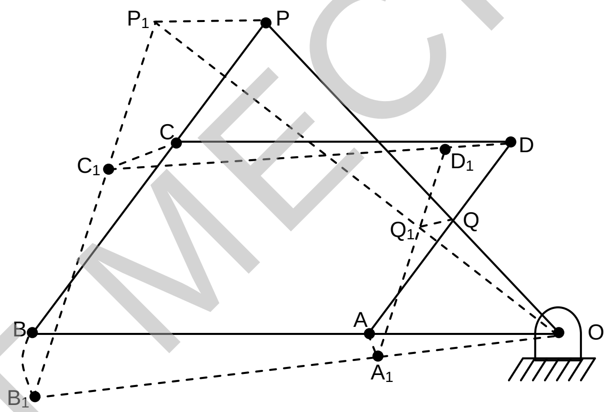


Figure 1.33 : Pantograph Mechanism

Suppose a point *Q* on the link *AD* moves to position Q_1 by rotating the link *OAB* downward. Now all the links and the joints will move to the new positions : *A* to A_1, *B* to B_1, *C* to C_1, *D* to D_1 and *P* to P_1 and the new configuration of the mechanism will be as shown by dotted lines. The movement of *Q* (QQ_1) will stand enlarged to PP_1 in a definite ratio and in the same form as proved below :

Triangles *OAQ* and *OBP* are similar. Therefore,

$$\frac{OA}{OB} = \frac{OQ}{OP}$$

In the dotted position of the mechanism when *Q* has moved to position Q_1 and correspondingly *P* to P_1, triangles OA_1Q_1 and OB_1P_1 are also similar since length of the links remain unchanged.

$$\frac{OA_1}{OB_1} = \frac{OQ_1}{OP_1}$$

But $$OB_1 = OB$$

$$OA_1 = OA$$

$\therefore$ $$\frac{OA}{OB} = \frac{OQ_1}{OP_1}$$

$$\frac{OQ}{OP} = \frac{QQ_1}{OP_1}$$

As such triangle OQQ_1 and OPP_1 are similar, and PP_1 and QQ_1 are parallel and further,

$$\frac{PP_1}{OP} = \frac{QQ_1}{OQ}$$

$$PP_1 = OQ_1 \times \frac{QQ_1}{OQ}$$

$$= QQ_1 \times \frac{OB}{OA}$$

Therefore, PP_1 is a copied curve at enlarged scale.

1.11.2 Straight Line Motion Mechanisms

A mechanism built in such a manner that a particular point in it is constrained to trace a straight line path within the possible limits of motion, is known as a *straight line motion mechanism*.

The Scott Russel Mechanism

This mechanism is shown in Figure 1.34. It consists of a crank *OC*, connecting rod *CP*, and a slider block *P* which is constrained to move in a horizontal straight line passing through *O*. The connecting rod *PC* is extended to *Q* such that

$$PC = CQ = CO$$

It will be proved that for all horizontal movements of the slider *P*, the locus of point *Q* will be a straight line perpendicular to the line *OP*.

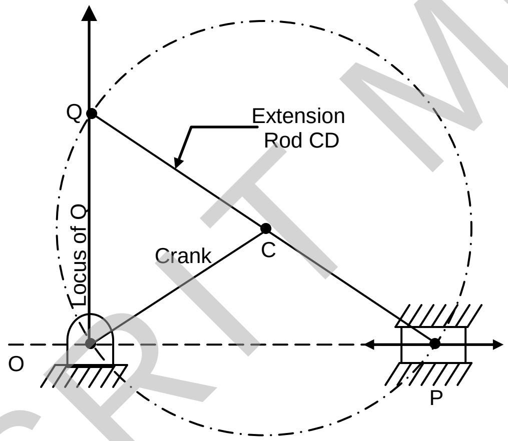


Figure 1.34 : Scott Russel Mechanism

Draw a circle of diameter *PQ* as shown. It is will known that diameter of a circle always subtends a right angle or any point on the circle. Thus, at point *O*, the angle *QOP* is a right angle. For any position of *P*, the line connecting *O* with *P* will always be horizontal. Therefore, line joining the corresponding position of *Q* with *O* will always a straight line perpendicular to *OP*. Thus, the locus of point *Q* will be straight line perpendicular to *OP*. Thus, a horizontal straight line motion of slider block *P* will enable point *Q* to generate a vertical straight line, both passing through *O*.

Generated Straight Line Motion Mechanisms

Principle

The principle of working of an accurate straight line mechanism is based upon the simple geometric property that the inverse of a circle with reference to a pole on the circle is a straight line. Thus, referring to Figure 1.35, if straight line *OAB* always passes through a fixed pole *O* and the points *A* and *B* move in such a manner that : $OA \times OB$ = constant, then the end *B* is said to trace an inverse line to the locus of A moving on the circle of diameter *OC*. Stated otherwise if *O* be a point on the circumference of a circle diameter *OP*, *OA* by any chord, and *B* is a point on *OA* produced, such that $OA \times OB = a$ constant, then the locus of a point *B* will be a straight line perpendicular to the diameter *OP*. All this is proves as follows :

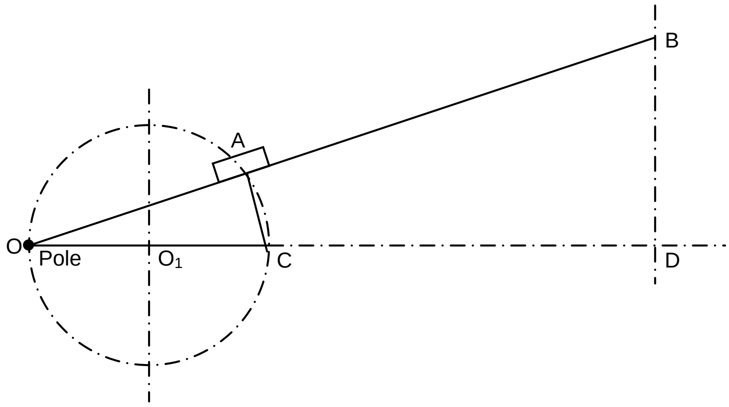


Figure 1.35 : Straight Line Motion Mechanism

Draw a horizontal line from *O*. From *A* draw a line perpendicular to *OA* cutting the horizontal at *C*. *OC* is the diameter of the circle on which the point *A* will move about *O* such that $OA \times OB$ remains constant.

Now, Δ*s* *OAC* and *ODB* are similar.

Therefore, $$\frac{OA}{OC} = \frac{OD}{OB}$$

$$\Rightarrow \qquad OD = \frac{OA \times OB}{OC}$$

But *OC* is constant and so that if the product $OA \times OB$ is constant, *OD* will be constant, or the position of the perpendicular from *B* to *OC* produced is fixed. This is possible only if the point *B* moves along a straight path *BD* which is perpendicular to *OC* produced.

A number of mechanisms have been innovated to connect *O*, *B* and *A* in such a way as to satisfy the above condition. Two of these are given as follows :

The Peaucellier Mechanism

The mechanism consists of isosceles four bar chain *OKBM* (Figure 1.36). Additional links *AK* and *AM* from, a rhombus *AKBM*. *A* is constrained to move on a circular path by the radius bar *CA* which is equal to the length of the fixed link *OC*.

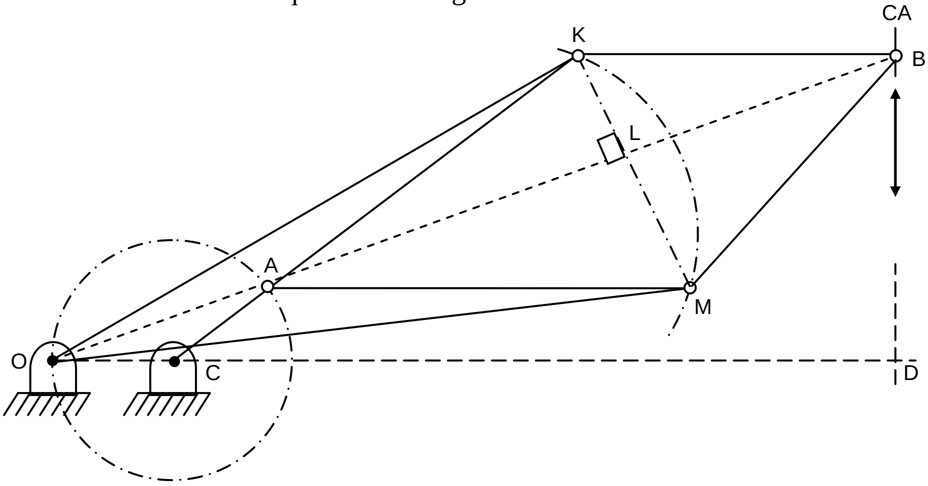


Figure 1.36 : The Peaucellier Mechanism

From the geometry of the figure, it follows that

$$OA \times OB = (OL - AL)\,(OL + LB) = OL^2 - AL^2 \qquad [\because\ AL = LB]$$

$$= (OK^2 - KL^2) - (AK^2 - KL^2) = OK^2 - AK^2 = \text{constant}$$

Hence, $OA \times OB$ is constant for a given configuration and B traces a straight path perpendicular to OC produced.

The Hart's Mechanism

This is also known as *crossed parallelogram mechanism*. It is an application of four-bar chain. *PSQR* is a four-bar chain in which

$SP = QR$

and $SQ = PR$ (Figure 1.37)

On three links *SP*, *SQ* and *PQ*, then it can be proved that for any configuration of the mechanism :

$OA \times OB$ = Constant

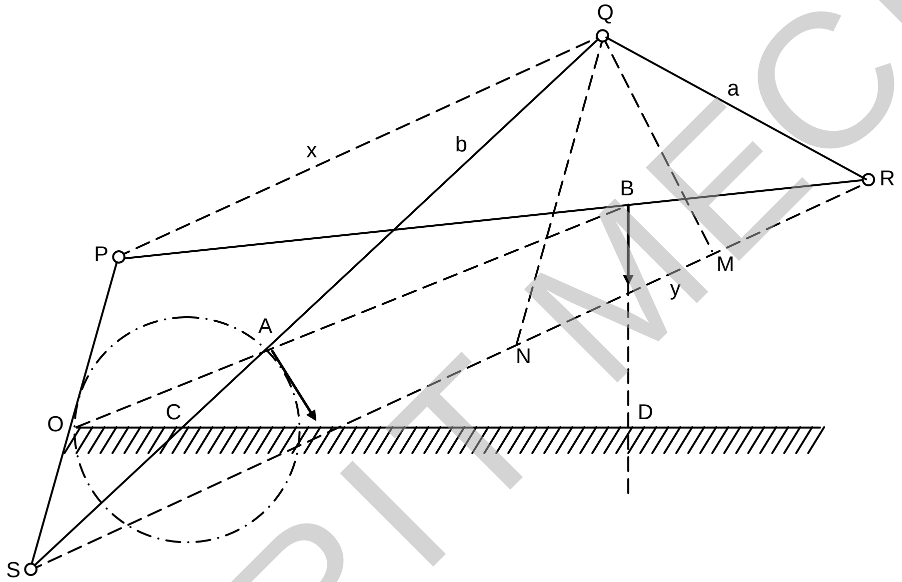


Figure 1.37 : Hart's Mechanism

The proof is given as follows :

Let $SP = QR = a$

$SQ = PR = b$

$PQ = x$

and $SR = y$

Then $OA \times OB = \dfrac{OS}{a} \times x \times \dfrac{OP}{a} \times y = xy \times \text{constant}$

But $x = SM - NM\ [QK \perp SR]$

$y = SM + NM\ [QN \parallel PS]$

$xy = (SM^2 - NM^2)$

$= (b^2 - QM^2) - (a^2 - QM^2) = (b^2 - a^2)$

Hence, $OA \times OB = xy \times \text{constant} = \text{constant}$

It is therefore, concluded if the mechanism is pivoted at *Q* as a fixed point and the point *A* is constrained to move on a circle through *O* the point *B* will trace a straight line perpendicular to the radius *OC* produced.

Approximate Straight-line Mechanisms

With the four-bar chain a large number of mechanisms can be devised which give a path which is approximately straight line. These are given as follows :

The Watt Mechanism

OABO′ is the mechanism used for Watt for obtaining approximate straight line motion (Figure 1.38). It consists of three links : *OA* pivoted at *O*. *O′B* pivoted at *O′* and both connected by link *AB*. A point *P* can be found on the link *AB* which will have an approximate straight line motion over a limited range of the mechanism. Suppose in the mean position link *OA* and *O′B* are in the horizontal position an *OA′* and *O′B′* are the lower limits of movement of these two links such that the configuration is *OA′B′O′*. Let *I* be the instantaneous centre of the coupler link *A′B′*, which is obtained by producing *OA′* and *B′O′* to meet at *I*. From *I* draw a horizontal line to meet *A′B′* at *P′*. This point *P′*, at the instant, will move vertically.

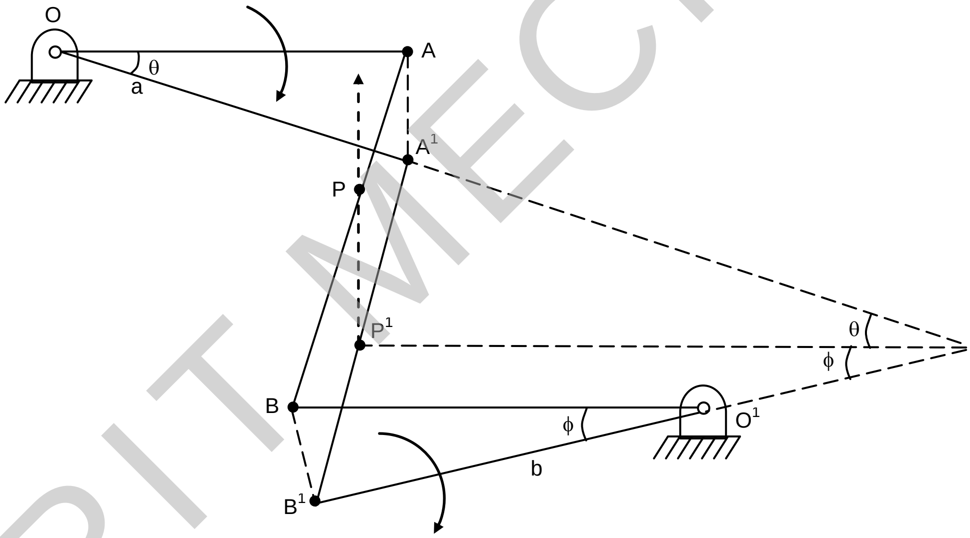


Figure 1.38 : The Watts Mechanism

Considering angles θ and ϕ being exceedingly small, as an approximation,

$$\frac{A'P'}{B'P'} = \frac{\theta}{\phi} = \frac{AA'}{a} + \frac{BB'}{b} = \frac{b}{a}$$

Where *a* and *b* are the lengths of *OA* and *O′B* respectively. Since, both *OA* and *OB* are horizontal in the mean or mid-position ever point in the mechanism then moves vertically.

Hence, if *P* divides *AB* in the ratio

$$AP : BP = b : a$$

then *P* will trace a straight line path for a small range of movement on either side of the mean position of *AB*.

The Grass-hopper Mechanism

It is shown in Figure 1.39. It is a modification of Scott-Russel mechanism. It consists of crank *OC* pivoted at *O*, link *O′P* pivoted at *O′* and a link *PCR* as shown. It is, in fact, a laid out four-bar mechanism. Line joining *O* and *P* is horizontal in middle position of the mechanism. The lengths of the link are so fixed such that :

$$OC = \frac{(CP)^2}{CR}$$

If this condition is satisfied, it is found that for a small angular displacement of the link *O′P*, the point *R* on the link *PCR* will trace approximately a straight line path, perpendicular to line *PQ*.

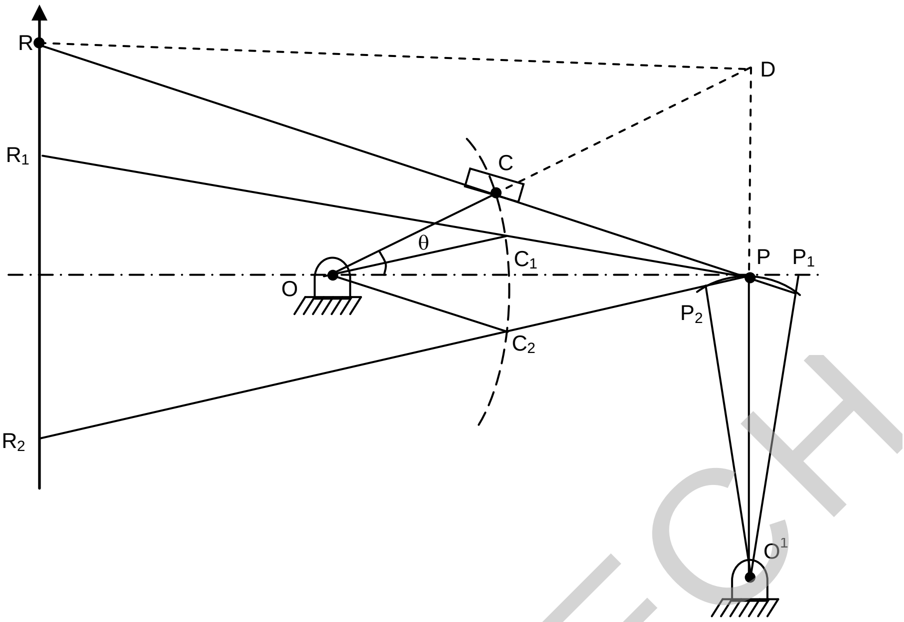


Figure 1.39 : Grass-hopper Mechanism

In Figure 1.39, the positions both of *P* and *R* have been shown for three different configuration of links. It may be noted that the pin at *C* is slidable along with link *RP* such that at each position the above equation is satisfied.

Robert's Mechanism

This is also a four-bar chain *ABCD* in which links *AD* = *BC* (Figure 1.40). The tracing point *P* is obtained by intersection of the right bisector of the couple *CD* and a perpendicular on the horizontal from the instantaneous centre *I*. Thus, an additional link *E* is connected to the coupler link *BC* and the path of point *P* is approximately horizontal in this Robert's mechanism.

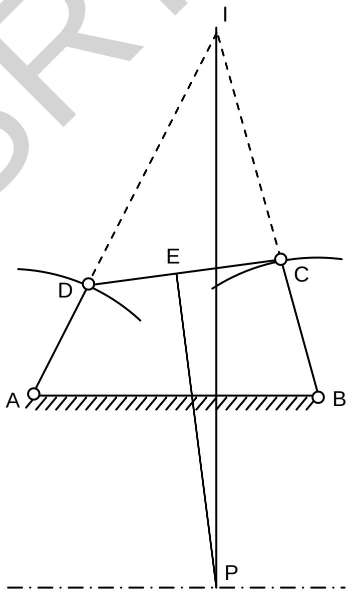


Figure 1.40 : Robert's Mechanism

Tchebicheff's Mechanism

This consists of four-bar chain in which two links *AB* and *CD* of equal length cross each other; the tracing point P lies in the middle of the connecting link *BC* (Figure 1.41). The proportions of the links are usually such that *P* is directly above *A* or *D* in the extreme position of the mechanism, i.e. when *CB* lies along *AB* or when *CB* lies along *CD*. It can be shown that in these circumstances the tracing point *P* will lies on a straight line parallel to *AD* in the two extreme positions and in the mid position if

$$BC : AD : AB \ :: \ 1 : 2 : 2.5$$

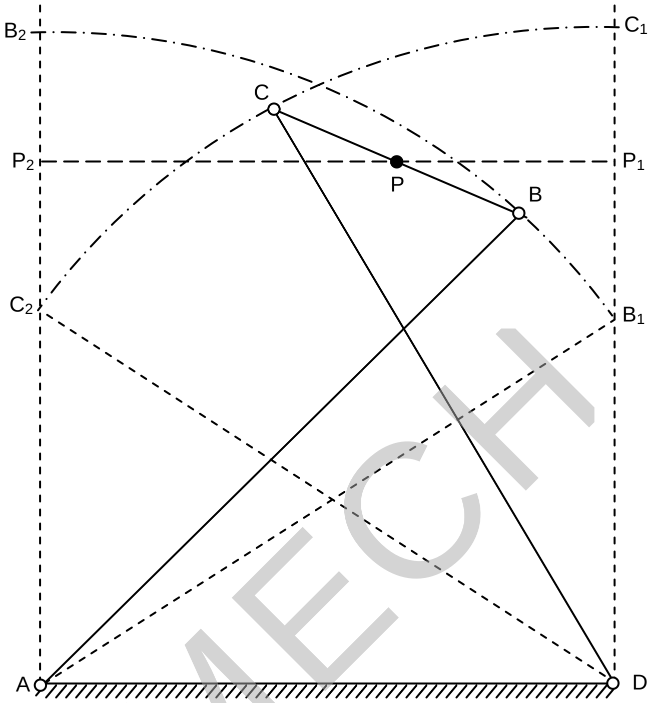


Figure 1.41 : Tchebicheff's Mechanism

SAQ 6

Which mechanisms are used for

(a) exact straight line, and

(b) approximate straight line.

1.11.3 Automobile Steering Gear

In an automobile vehicle the relative motion between its wheels and the road surface should be one of pure rolling. To satisfy this condition, the steering gear should be so designed that when the vehicle is moving along a curved path, the paths of points of contact of each wheel with the road surface should be concentric circular arcs. The steering or turning of a vehicle to one side or the other is accomplished by turning the axis of rotation of the front two wheels. Each front wheel has a separate short axle of rotation, known as *stub axle* such as *AB* and *CD*. These sub axles are pivoted to the chassis of the vehicle. *To satisfy the condition of pure rolling during turning, the design of the steering gear should be such that at any instant while turning the axes of rotation of the front and the rear wheels must intersect at one point which is known as instantaneous centre denoted by I*. The whole vehicle is assumed to be revolving about this point at the instant considered. In Figure 1.42, *AB* and *CD* are the short axles of the front wheel and *EF* for the rear wheels.

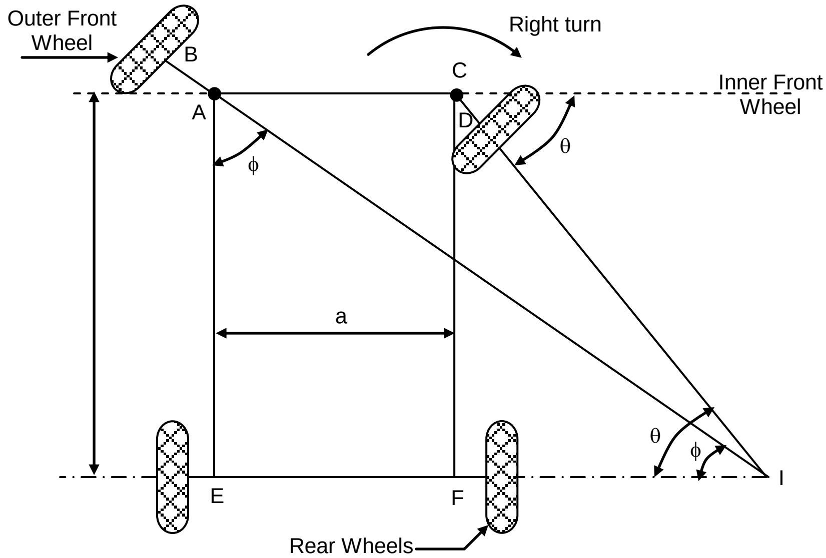


Figure 1.42 : Automobile Steering Gear

While turning to the right side, axes of the front and the rear wheels meet at *I*.

Suppose θ = The angle by which the inner wheel is turned;

ϕ = The angle by which the outer wheel is turned;

A = Distance between the points of the front axles; and

l = Wheel base *AE*.

As may be seen from the geometry of the Figure 1.42, the angle of turn θ of the inner front wheel is always more than the angle of turn ϕ of the outer front wheel.

From Figure 1.42,

$$a = AC = EF = EI - FI = l\,(\cot\phi - \cot\theta)$$

$$\therefore \qquad a = l\,(\cot\phi - \cot\theta)$$

This is the fundamental equation of steering. If this equation is satisfied in a vehicle, it is assured that the vehicle while taking a turn of any angle would not slip but would have pure rolling motion between its wheels and the road surface.

Types of Steering Gears

There are mainly two types of steering gear mechanisms :

(a) Davis steering gear,

(b) Ackerman's steering gear,

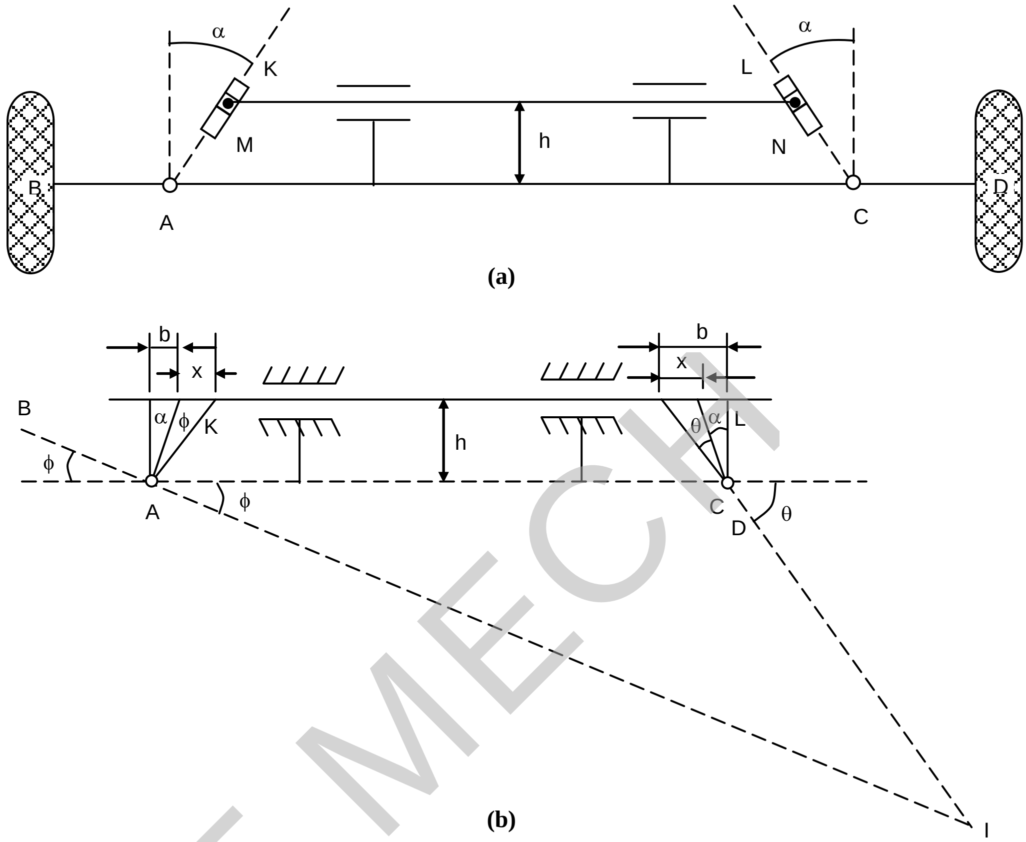
Both these mechanisms are described separately as follows :

Davis Steering Gear

This steering gear mechanism is shown in Figure 1.43(a). It consists of the main axle *AC* having a parallel bar *MN* at a distance *h*. The steering is accomplished by sliding bar *MN* within the guides (shown) either to left or to the right hand side. *KAB* and *LCD* are two bell-crank levers pivoted with the main axle at *A* and *C* respectively such that $\angle BAK$ and $\angle DCL$ remain always constant. Arms *AK* and *CL* have been provided with slots and these house die-blocks *M* and *N*. With the movement of bar *MN* at the fixed height, it is the slotted arms *AK* and *CL* which side relative to the die-blocks *M* and *N*.

In Figure 1.43(a), the vehicle has been shown as moving in a straight path and both the slotted arms are inclined at an angle α as shown.

Now suppose, for giving a turn to the right hand side, the base *MN* is moved to the right side by distance *x*. The bell-crank levers will change to the positions shown by dotted lines in Figure 1.43(b). The angle turned by the inner wheel and the outer wheels are θ and φ respectively. The arms *BA* and *CD* when produced will meet say at *I*, which will be the instantaneous centre.



(a)

(b)

Figure 1.43 : Davis Steering Gear

Suppose $2b$ = Difference between *AC* and *MN*, and

α = Angle *AK* and *CL* make with verticals in normal position.

$$\tan \alpha = \frac{b}{h} \qquad \ldots (1.1(a))$$

$$\tan (\alpha + \phi) = \frac{(b + x)}{h} \qquad \text{[considering point } A\text{]} \qquad \ldots (1.2(b))$$

$$\tan (\alpha - \theta) = \frac{(b - x)}{h} \qquad \text{[considering point } C\text{]} \qquad \ldots (1.2(c))$$

Now,
$$\tan (\alpha + \phi) = \frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \tan \phi}$$

or
$$\frac{b + x}{h} = \frac{\frac{b}{h} + \tan \phi}{1 - \frac{b}{h} \times \tan \phi}$$

$$b + h \tan \phi = (b + x) \frac{(h - b \tan \phi)}{h}$$

$$hb + h^2 \tan \phi = (b + x)(h - b \tan \phi)$$

$$= bh + hx - b^2 \tan \phi + xb \tan \phi$$

$$h^2 \tan\phi + b^2 \tan\phi + xb\tan\phi = bh + hx - bh = hx$$

$$\tan\phi = (h^2 + b^2 + xb) = hx$$

$$\tan\phi = \frac{hx}{(h^2 + b^2 + xb)} \qquad \text{. . . (1.2(d))}$$

Similarly, $$\tan(\alpha - \theta) = \frac{\tan\alpha - \tan\theta}{1 + \tan\alpha\tan\theta} = \frac{b - x}{h}$$

Studying for tan α and simplifying :

$$\tan\theta = \frac{hx}{(h^2 + b^2 - xb)} \qquad \text{. . . (1.2(e))}$$

After obtaining the expressions for tan ϕ and tan θ, let us not take up the fundamental equation of steering :

$$\cot\phi - \cot\theta = \frac{h^2 + b^2 + xb}{hx} - \frac{(h^2 + b^2 - xb)}{xh}$$

$$\Rightarrow \qquad \cot\phi - \cot\theta = \frac{2b}{h} = 2\tan\alpha \qquad \text{. . . (1.2(f))}$$

But for correct steering,

$$\boldsymbol{\cot\phi - \cot\theta = \frac{a}{l}}$$

$$\Rightarrow \qquad 2\tan\alpha = \frac{a}{l}$$

$$\boldsymbol{\tan\alpha = \frac{a}{2l}} \qquad \text{. . . (1.2(g))}$$

The ratio $\frac{a}{l}$ varies from 0.4 to 0.5 and correspondingly α to 14.1°.

The demerits of the Davis gear are that due to number of sliding pairs, friction is high and this causes wear and tear at contact surfaces rapidly, resulting in in-accuracy of its working.

Ackermann Steering Gear

The mechanism is shown in Figure 1.44(a). This is simpler than that of the Davis steering gear system. It is based upon four-bar chain. The two opposite links *AC* and *MN* are unequal; *AC* being longer than *MN*. The other two opposite links *AM* and *CN* are equal in length. When the vehicle is moving on a straight path link *AC* and *MN* are parallel to each other. The shorter links *AM* and *CN* are inclined at angle α to the longitudinal axis of the vehicle as shown. *AB* and *CD* are *stub axles but* integral part of *AM* and *CN* such that *BAM* and *DCN* are bell-crank levers pivoted at *A* and *C*. Link *AM* and *CN* are known as *track arms* and the link *MN* as track rod. The track rod is moved towards left or right hand sides for steering. For steering a vehicle on right hand side, link *NM* is moved towards left hand side with the result that the link *CN* turns clockwise. Thus, the angle α is increased and that on the other side, it is decreased. From the arrangement of the links it is clear that the link *CN* or the inner wheel will turn by an angle θ which is more than the angle of turn of the outer wheel or the link *AM*.

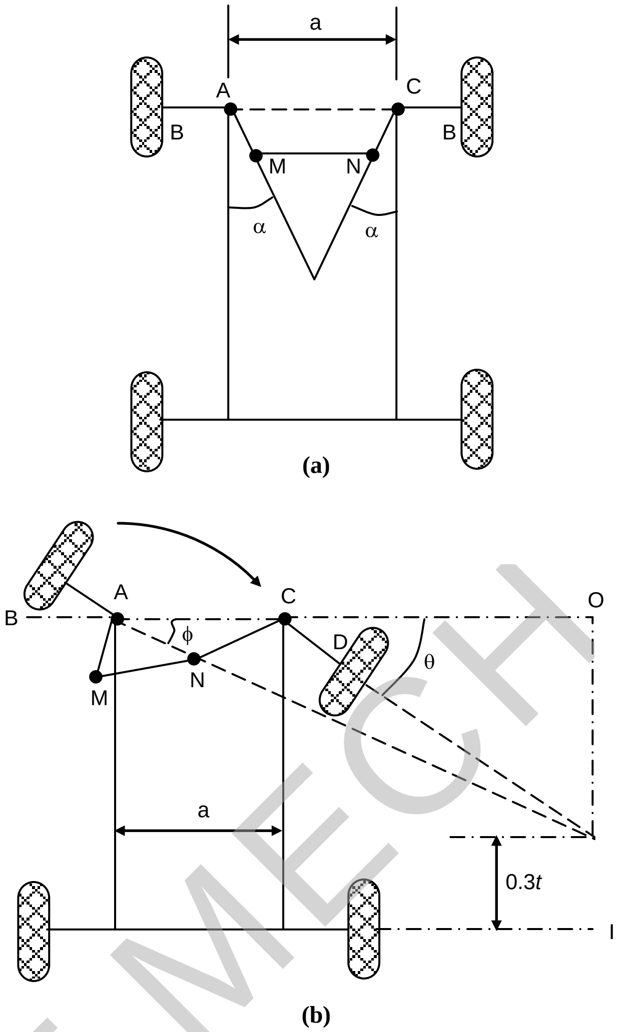


Figure 1.44 : Ackermann Steering Gear

To satisfy the basic equation of steering :

$$\cot \phi - \cot \theta = \frac{a}{l} ,$$

the links *AM* and *MN* are suitably proportioned and the angle α is suitable selected. In a given automobile, with known dimensions of the four-bar links, angle α is known. For different angle of turn θ, the corresponding value of ϕ are noted. This is done by actually drawing the mechanism to a scale. Thus, for different values of θ, the corresponding value of ϕ and $(\cot \phi - \cot \theta)$ are tabulated.

As given above, for correct steering,

$$\cot \phi - \cot \theta = \frac{a}{l}$$

For approximately correct steering, value of $\frac{a}{l}$ should be between 0.4 and 0.5.

Generally, it is 0.455. In fact, there are three values of θ which give correct steering; one when $\theta = 0$, second and third for corresponding turning to the right and the left hand.

Now there are two values of ϕ corresponding to given values of θ. The value actually determined graphically by drawing the mechanism and tabulating ϕ corresponding to different values of θ is known as *actual* ϕ

or ϕ_a. But the value of ϕ obtained from the fundamental equation
$\cot \phi - \cot \theta = \frac{a}{l}$ corresponding to different values of θ is known as *correct* ϕ or ϕ_c. On making comparison between the two values it is found that ϕ_a is higher than ϕ_c for small values of θ, ϕ_a are lower than ϕ_c. The difference is negligible for small value of θ but for large values of θ, it is substantial. This would reduce the life of the tyres due to greater wear on account of slipping but then for large value of θ, the vehicle takes a sharper turn as such its speed is reduced and accordingly the wear is also reduced. Thus, the greater difference between ϕ_a and ϕ_c for large value of θ will not matter much.

As a matter of fact the position for correct steering is only an arbitrary condition. In Ackermann steering, for keeping the value of angle θ on the lower side, the instantaneous centre of the front wheels does not lie on the line of axis of the rear wheel as shown in Figure 1.44(b).

1.11.4 Hook's Joint or Universal Coupling

It is shown in Figure 1.45, it is also known as *universal joint*. It is used for connecting two shafts whose axes are non-parallel but intersecting as shown in Figure 1.45. Both the shafts, driving and the driven, are forked at their ends. Each fork provides for two bearings for the respective arms of the cross. The cross has two mutually perpendicular arms. In fact, the cross acts as an intermediate link between the two shafts. In the figure, the driven shaft has been shown as inclined at an angle α with the driving shaft.

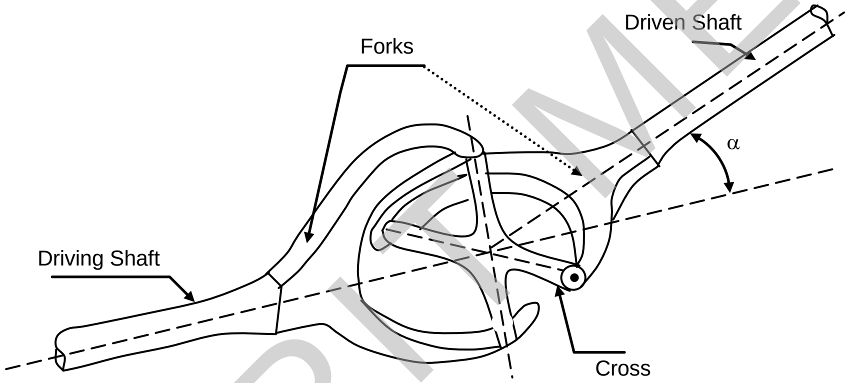


Figure 1.45 : Hook's Joint

The Hook's joint is generally found being used for transmission of motion from the gear box to the back axle of automobile and in the transmission of drive to the spindles in a multi-spindle drilling machines. There are host of other applications of the Hook's joint where motion is required to be transmitted in non-parallel shafts with their axes intersecting.

Figure 1.46(a) gives the end of the driving shafts. *AB* and *CD* are the mutually perpendicular arms of the cross in the initial position. Arm *AB* is of the driving shaft and *CD* for the driven shaft. The plan of rotation of the driving shaft and its arm *AB* will be represented in the plane of the paper in elevation.

In Figure 1.46(b), i.e. in the plan the direction of driving and driven shaft and that of the cross arms are given. The driven shaft is inclined at an angle α with the axis of the driving shaft.

PP gives the direction of the arm connected to the driving shaft and *QQ* gives the direction of the arm connected to the driven shaft. In fact, the traces *PP* and *QQ* give the plane of rotation of the arms of the cross, as seen in the plan view.

Now, suppose, the driving shaft turns by angle θ. The arm *AB* will also turn by θ and will take the position A_1B_1 as shown in the elevation. Suppose, correspondingly the driven shaft and its arm *CD* are rotated by ϕ. The new position of *CD* is C_2O. With the rotation of AB by θ, it is the projection C_1D_1 of CD which will rotate through angle θ. OC_1 is the projection of OC and its sure length is given by OC_2 and accordingly the angle of rotation φ of the arm CD of the driven shaft, is known.

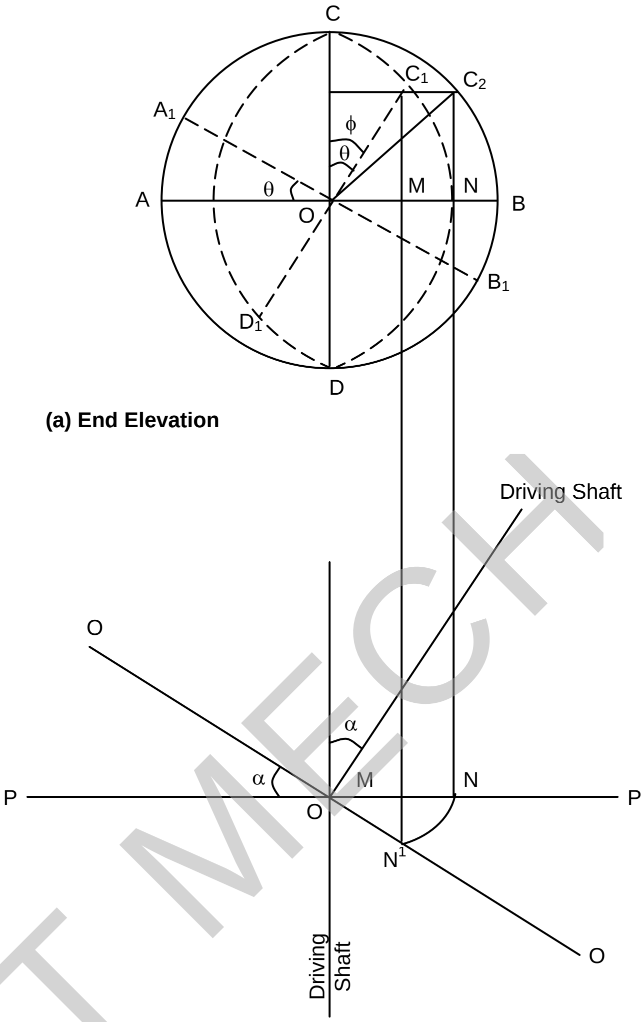


Figure 1.46

Ratio between φ and θ

As given above,

θ = The angle through which the driving shaft is rotated, and

φ = The corresponding angle through which the driven shaft is rotated.

Refer Figures 1.46(a) and (b).

$$\tan\theta = \frac{OM}{OC_1} = \frac{OM}{NC_2} \quad \ldots (1.3)$$

$$\tan\phi = \frac{ON}{NC_2}$$

$$\frac{\tan\theta}{\tan\phi} = \frac{OM}{NC_2} \times \frac{NC_2}{ON} = \frac{OM}{ON} \quad \ldots (1.4)$$

But from Figure 1.46(b)

$$\frac{OM}{ON} = \cos\alpha$$

$$\Rightarrow \quad \frac{\tan\phi}{\tan\theta} = \frac{1}{\cos\alpha} \quad \ldots (1.5)$$

Ratio between Speed of Driven and Driving Shafts

ω = Angular speed of the driving shaft, and

ω_1 = Angular speed of the driven shaft.

$$\omega = \frac{d\theta}{dt}$$

$$\omega_1 = \frac{d\phi}{dt}$$

By Eq. (1.4)

$$\frac{\tan\phi}{\tan\theta} = \frac{1}{\cos\alpha}$$

$$\tan\theta = \cos\alpha \tan\phi$$

Differentiating both sides,

$$\sec^2\theta \frac{d\theta}{dt} = \cos\alpha \sec^2\phi \frac{d\phi}{dt}$$

$$\sec^2\theta \times \omega = \cos\alpha \sec^2\phi \times \omega_1$$

$$\frac{\omega}{\omega_1} = \frac{\cos\alpha \sec^2\phi}{\sec^2\theta} = \cos\alpha \times \cos^2\theta \times \sec^2\phi \qquad \ldots (1.4(a))$$

But $\sec^2\theta = 1 + \tan^2\phi$

From Eq. (1.4),

$$\tan\phi = \frac{\tan\theta}{\cos\alpha}$$

$$\tan^2\phi = \frac{\tan^2\theta}{\cos\alpha}$$

$$\therefore \quad \sec^2\phi = 1 + \frac{\tan^2\theta}{\cos^2\alpha} = 1 + \frac{\sin^2\theta}{\cos^2\theta\cos^2\alpha}$$

$$= \frac{\cos^2\theta \times \cos^2\alpha + \sin^2\theta}{\cos^2\theta\cos^2\alpha}$$

$$= \frac{\cos^2\theta\,(1 - \sin^2\alpha) + 1 - \cos^2\theta}{\cos^2\theta\cos^2\alpha}$$

$$= \frac{\cos^2\theta - \cos^2\theta\sin^2\alpha + 1 - \cos^2\theta}{\cos^2\theta\cos^2\alpha}$$

Hence, $$\sec^2\phi = \frac{1 - \cos^2\theta\sin^2\alpha}{\cos^2\theta\cos^2\alpha}$$

But as per Eq. (1.4(a)),

$$\frac{\omega}{\omega_1} = \cos\alpha\cos^2\theta\sec^2\phi$$

Substituting for $\sec^2 \phi$

$$\therefore \qquad \frac{\omega}{\omega_1} = \frac{(1 - \cos^2 \theta \sin^2 \alpha) \cos^2 \alpha \sec^2 \theta}{\sec^2 \theta \cos^2 \alpha}$$

$$= \frac{1 - \cos^2 \theta \sin^2 \alpha}{\cos \alpha}$$

$$= \frac{\text{Speed of driven}}{\text{Speed of driver}}$$

$$= \boldsymbol{\frac{\omega}{\omega_1} = \frac{\cos \theta}{1 - \cos^2 \theta \cos^2 \alpha}} \qquad \ldots (1.5)$$

Condition for Maximum and Minimum Speed Ratio

For a given value of α :

$\frac{\omega}{\omega_1}$ will be a maximum

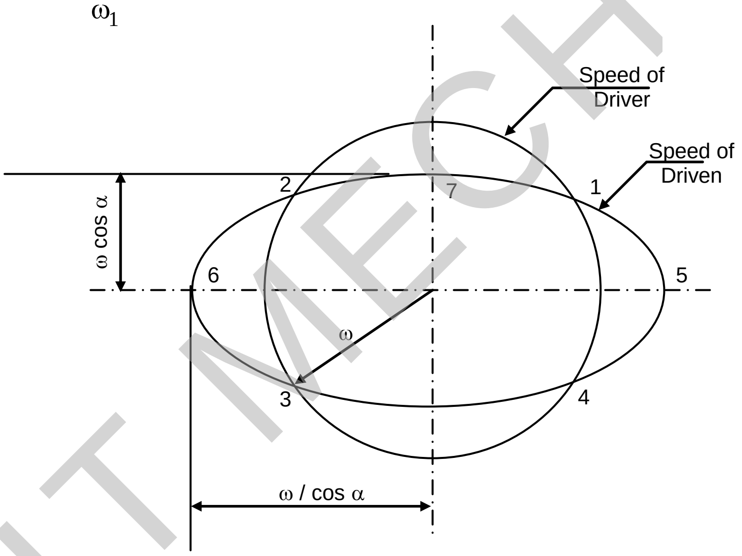


Figure 1.47 : Polar Velocity Diagram

when in the equation :

$$\frac{\omega}{\omega_1} = \frac{\cos \alpha}{1 - \cos^2 \theta \sin^2 \alpha} ;$$

The denominator $(1 - \cos^2 \theta \sin^2 \alpha)$ is minimum, i.e.

$$\cos^2 \theta = 1 \text{ or when } \cos \theta = \pm 1$$

or when $\theta = 0$ or 180^o, corresponding to points 5 and 6 in Figure 1.47 and the expression for the *maximum speed ratio* would be

$$\frac{\omega}{\omega_1} = \frac{\cos \alpha}{1 - \sin^2 \alpha} = \frac{1}{\cos \alpha} \qquad \ldots (1.6)$$

or $\qquad \boldsymbol{\omega_1 = \frac{\omega}{\cos \alpha}}$ [Represented by points 5 and 6 in Figure 1.47] . . . (1.7)

Similarly for a given angle α, $\frac{\omega}{\omega_1}$ will be minimum when in the equation.

$$\frac{\omega}{\omega_1} = \frac{\cos \alpha}{1 - \cos^2 \theta \sin^2 \alpha}$$

the denominator is maximum.

It will be so when $\theta = 90^o$ or 270^o.

In that case $\dfrac{\omega}{\omega_1} = \cos\alpha$ . . . (1.8)

$\Rightarrow$ For maximum speed ratio, $\boldsymbol{\omega_1 = \omega \cos\alpha}$ . . . (1.9)

Represented by points 7 and 8 in Figure 1.47.

Condition for the Same Speed

$$\frac{\omega}{\omega_1} = \frac{\cos\alpha}{1 - \sin^2\alpha \cos^2\theta}$$

For $\dfrac{\omega}{\omega_1}$ to be unity

$$1 = \frac{\cos\alpha}{1 - \sin^2\alpha \cos^2\theta}$$

$$\cos\alpha = (1 - \sin^2\alpha \cos^2\theta)$$

$$\cos^2\theta = \frac{1 - \cos\alpha}{\sin^2\alpha} = \frac{(1 - \cos\alpha)}{(1 - \cos^2\alpha)} = \frac{1}{1 + \cos\alpha}$$

$\Rightarrow$ For the same speed,

$$\boldsymbol{\cos\theta = \pm\sqrt{\frac{1}{1 + \cos\alpha}}} \quad \text{. . . (1.10)}$$

Condition for Maximum Variation of Driven Speed

Maximum variation of speed of driven shaft

$$= \frac{(\omega_{1\,max} - \omega_{1\,min})}{\omega_{mean} \text{ of driven shaft}}$$

where $\omega_{mean} = \omega$

$$\text{Maximum variation} = \frac{\left(\dfrac{\omega}{\cos\alpha} - \omega\cos\alpha\right)}{\omega} = \frac{(1 - \cos^2\alpha)}{\cos\alpha}$$

$$= \frac{\sin^2\alpha}{\cos\alpha} = \tan\alpha \sin\alpha \quad \text{. . . (1.11)}$$

$\Rightarrow$ Maximum variation = $\tan\alpha \sin\alpha$

If α is small, $\tan\alpha = \alpha$ as well as $\sin\alpha = \alpha$

$\Rightarrow$ **Maximum variation ≈ $\boldsymbol{\alpha^2}$** if α is very small . . . (1.12)

Generally, the speed of the driving shaft is constant. As such it can be represented by a circle of radius ω. In that case the maximum and minimum speed of the driven shaft will be $\dfrac{\omega}{\cos\alpha}$ and $\omega\cos\alpha$ respectively, represented by an ellipse of major axis $\dfrac{\omega}{\cos\alpha}$ and minor axis $\omega\cos\alpha$. This is shown in Figure 1.47 which is known as polar velocity diagram.

Angular Acceleration of the Driven Shaft

That angular acceleration of the driven shaft is given by $\dfrac{d\omega_1}{dt}$.

$$\therefore \qquad \alpha_1 = \frac{d\omega_1}{dt} = \frac{d\omega_1}{d\theta} \times \frac{d\theta}{dt} = \omega \frac{d\omega_1}{d\theta} \qquad \text{. . . (1.13)}$$

But by Eq. (1.4),

$$\frac{\omega}{\omega_1} = \frac{1 - \cos^2 \theta \sin^2 \alpha}{\cos \alpha}$$

$$\omega_1 = \frac{\omega \cos \alpha}{1 - \cos^2 \theta \sin^2 \alpha}$$

$$\frac{d\omega_1}{d\theta} = \frac{-\omega \cos \alpha \sin^2 \alpha \sin 2\theta}{(1 - \cos^2 \theta \sin^2 \alpha)^2}$$

By Eq. (1.13), $\alpha_1 = \omega \dfrac{d\omega_1}{d\theta}$

$\Rightarrow$ Angular acceleration of driven shaft :

$$\boldsymbol{\alpha_1 = \frac{-\omega^2 \cos \alpha \sin^2 \sin 2\theta}{(1 - \cos^2 \theta \sin^2 \alpha)^2}} \qquad \text{. . . (1.14)}$$

For determining conditions for maximum acceleration, differentiate α_1, w.r.t. θ and equate it to zero. The resulting expression is, however, very complicated, and it will be found that the following expression which is derived from the exact expression by a simple approximation, gives results which are sufficiently close for most practical purpose.

$\Rightarrow$ For maximum α_1, $\boldsymbol{\cos 2\theta \approx \dfrac{2\sin^2 \alpha}{2 - \sin^2 \alpha}}$ . . . (1.15)

This equation gives the value of θ almost accurate upto a maximum value of α as 30^o. It should be noted that the angular acceleration of the driven shaft is a maximum when θ is approximate 45^o, 135^o, etc., i.e. when the arms of the cross are inclined at 45^o to the plane contacting the axes of the two shafts.

1.12 CAMS

In machines, particularly in typical textile and automatic machines, many parts need to be imparted different types of motion in a particular direction. This is accomplished by conversion of the available motion into the type of motion required. Change of circular motion to translatory (linear) motion of simple harmonic type and *vice-versa* and can be done by slider-crank mechanism as discussed previously. But now the question arises, what to do when circular or rotary motion is to be changed into linear motion of complex nature or into oscillatory motion. This job is well accomplished by a machine part of a mechanical member, known as *cam*.

1.12.1 Definition

A cam may be defined as a rotating, reciprocating or oscillating machine part, designed to impart reciprocating and oscillating motion to another mechanical part, called a *follower*.

A cam and follower have, usually, a line contact between them and as such they constitute a higher pair. The contact between them is maintained by an external force which is generally, provided by a spring or sometimes by the sufficient weight of the follower itself.

1.12.2 Classification of Cams

Cams are classified according to :

(a) Shape

(b) Follower movement

(c) Type of constraint of the follower

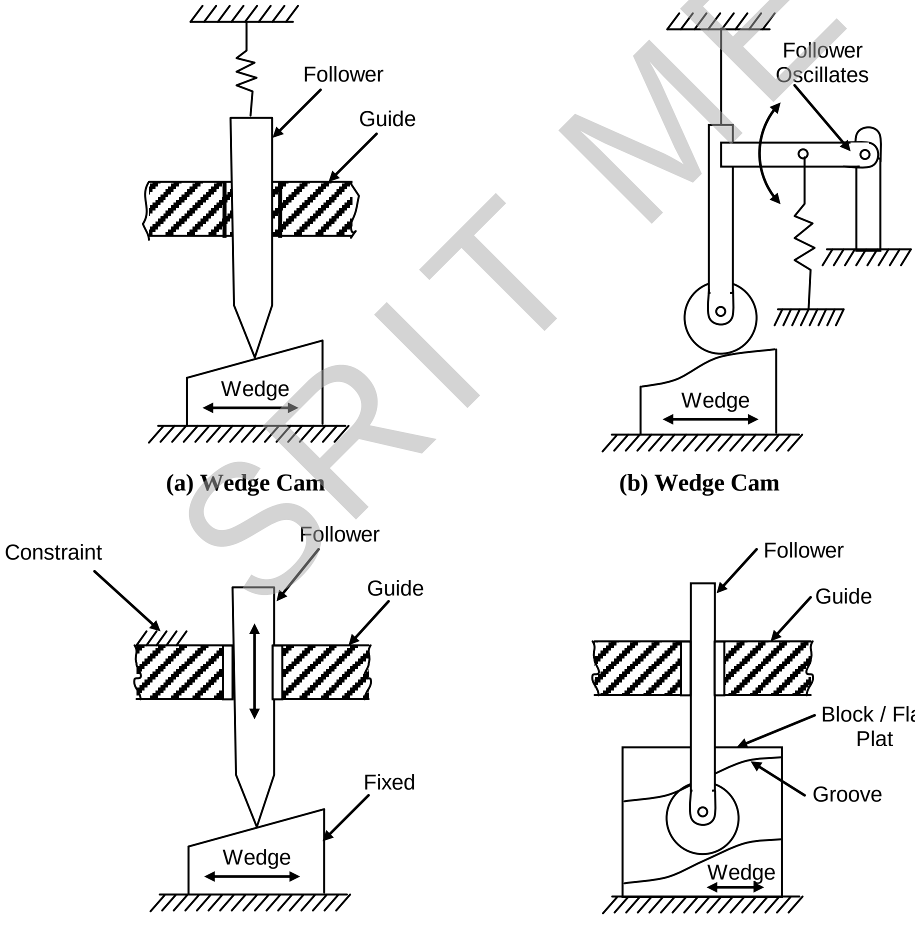
According to Shape

Wedge and Flat Cams

It is shown in Figures 1.48(a), (b), (c) and (d).

In Figure 1.48(a), on imparting horizontal translatory motion to wedge, the follower also translates but vertically in Figure 1.48(b), the wedge has curved surface at its top. The follower gets a oscillatory motion when a horizontal translatory motion is given to the wedge.

In Figure 1.48(c), the wedge is stationary, the guide is imparted translatory motion within the constraint provided. This results in translatory motion of the follower in Figure 1.48(d), instead of a wedge, a rectangular block or a flat plate with a groove is provided. When horizontal translatory motion is imparted to the block, the follower is constrained to have a vertical translatory motion.



(a) Wedge Cam **(b) Wedge Cam**

(c) Fixed Wedge Cam **(d) Flate Wedge Cam**

Figure 1.48

Further, there is no need to provide a spring in this case as in case (a) and (b). In this case the path of the groove, which causes motion to the follower, constrains the follower to move upward and downward.

Radial or Disc Cam

In radial or disc cams the shape of working surface (profile) is such that the followers reciprocate in a plane at right angles to the axis of the cam as shown in Figure 1.49(a). It is called as *radial cam* because the motion of the followers obtained is radial (Figure 1.49). A differently shaped radial cam is also shown in Figure 1.49(b).

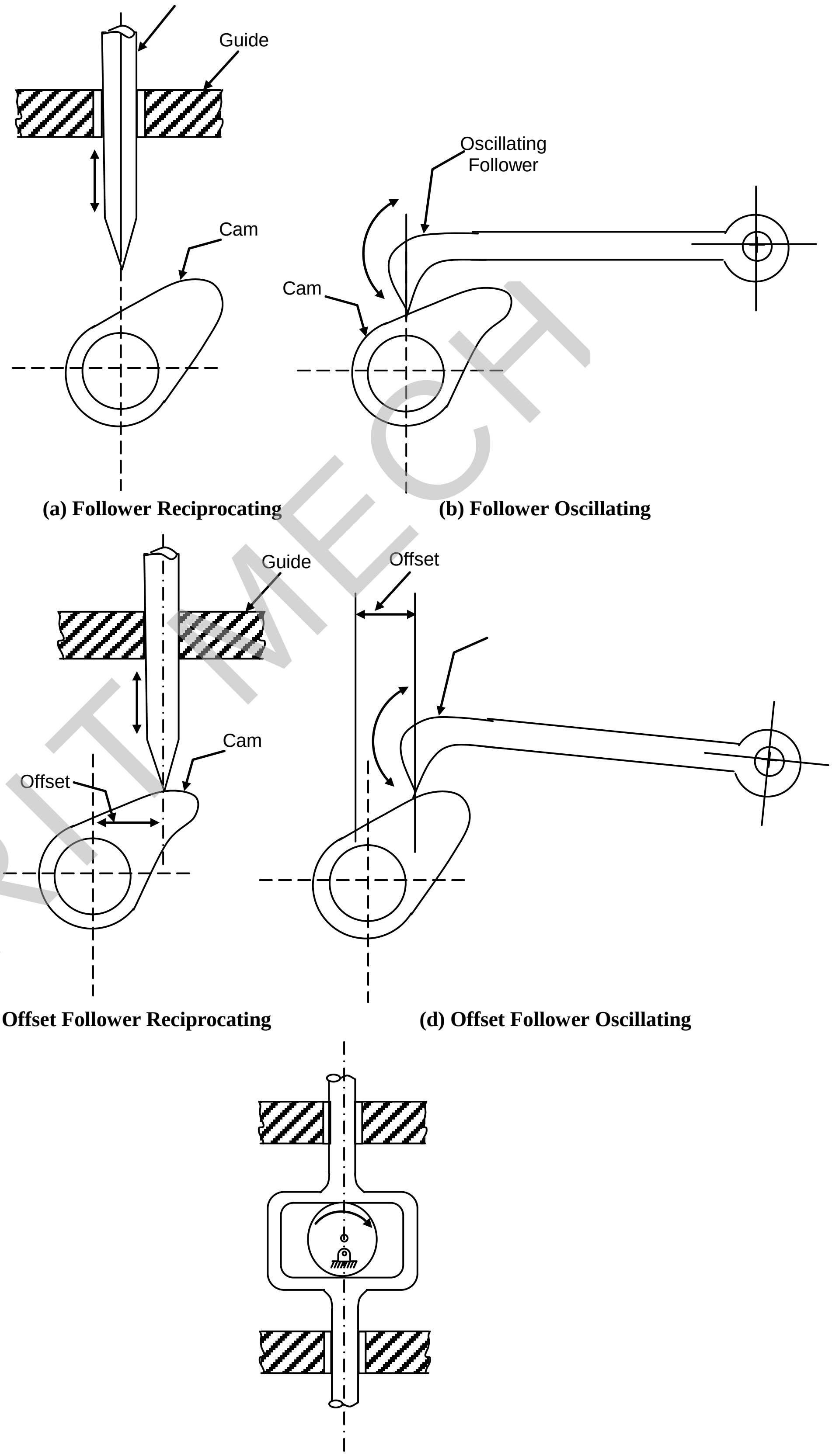
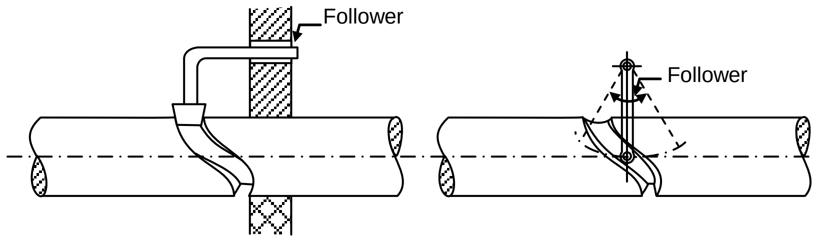


Figure 1.49 : Radial or Disc Cams

It is pointed out that the radial cams are very popular due to their simplicity and compactness,

Cylindrical Cams

Cylindrical cams have been shown in Figures 1.50(a) and (b). In Figure 1.50(a) the follower reciprocates whereas in Figure 1.50(b) the follower oscillates. Cylindrical cams are also known as *barrel* or *drum cams*.



(a) Follower Reciprocates **(b) Follower Oscillates**

Figure 1.50 : Cylindrical Cams

Spiral Cams

It is shown in Figure 1.51. The cam comprises of a plate on the face of which a groove of the form of a spiral is cut. The spiral groove is provided with teeth which mesh with pin gear follower.

This cam has a limited use because it has to reverse its direction to reset the position of the follower. *This cam has found its use in computers*.

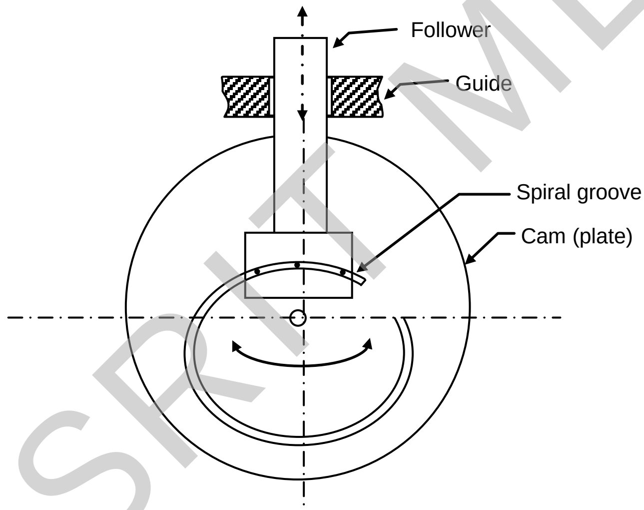


Figure 1.51 : Spiral Cam

Conjugate Cams

As the name implies, the cam comprises of two discs, keyed together and remain in constant touch with two rollers of a follower as shown in Figure 1.52.

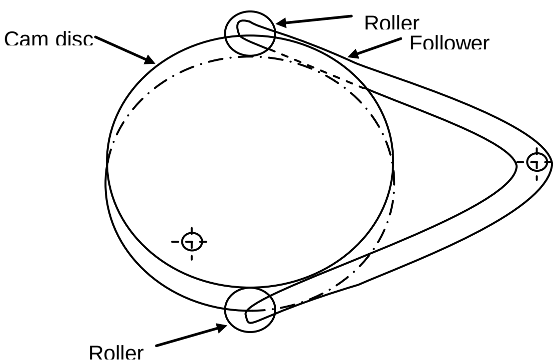


Figure 1.52 : Conjugate Cam

This cam is used where the requirement is of high dynamic load, low wear, low noise, high speed and better control of follower.

Globoidal Cams

This cam has two types of surfaces : *convex* and *concave*. A helical contour is cut on the circumference of the surface of rotation of the cam as shown in Figures 1.53(a) and (b). The end of the follower is constrained to move along the contour and then oscillatory motion is obtained. In this cam, a large angle of oscillation of the follower is obtained.

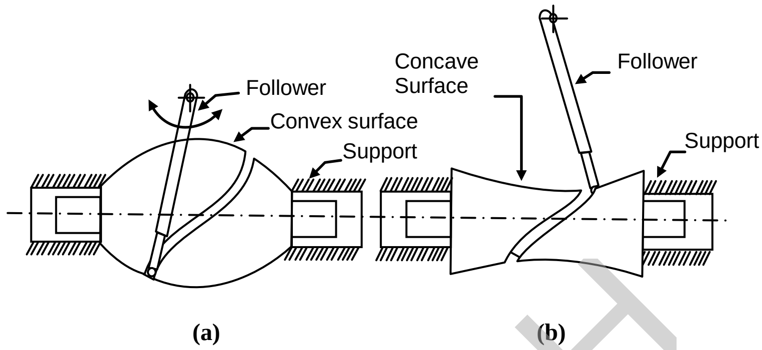


Figure 1.53 : Globoidal Cams

Spherical Cams

In this cam, as shown in Figure 1.54, the cam is of the shape of a sphere on the peripheral of which a helical groove is cut. The roller provided at the end of the follower rolls in the groove causing oscillatory motion to the follower in an axis perpendicular to the axis of rotation of the cam.

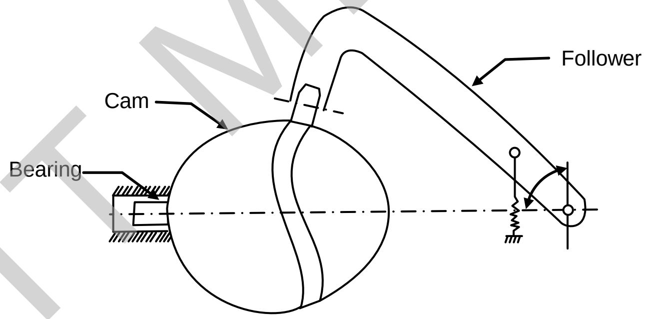


Figure 1.54 : Spherical Cam

According to Follower Movement

Rise-return-rise (*RRR*)

In this type of cam, its profile or contour is such that the cam rises, returns without rest or dwell, and without any dwell or rest, it again rises. Follower displacement and cam angle diagram for this type of cam is shown in Figure 1.55(a).

Dwell, Rise-return Dwell (*DRRD*)

In this type of cam after dwell, there is rise of the follower, then it returns to its original position and dwells for sometimes before again rising. Generally, this type of cam is commonly used. Its displacement cam angle diagram is shown in Figure 1.55(b).

Dwell-rise-dwell-return

It is the most widely used type of cam. In this, dwell is followed by rise. Then the follower remains stationary in the dwell provided and then returns to its original position [Figure 1.55(c)].

Dwell-rise-dwell

As may be seen in the follower-displacement *verses* cam angle diagram, shown in Figure 1.55(d) in this cam, the fall is sudden which necessities enormous amount of force for this to take place.

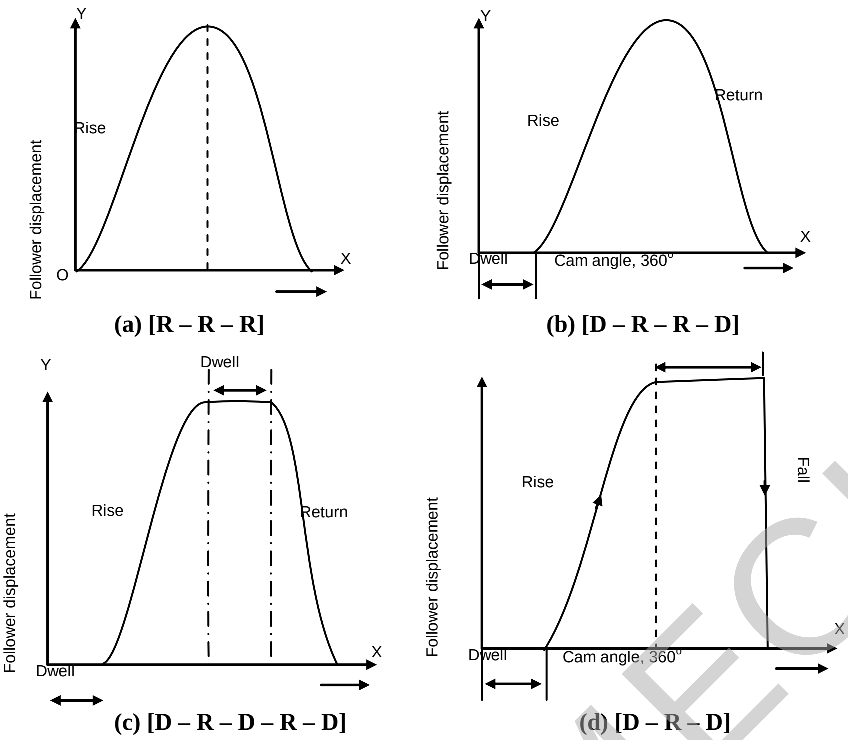


Figure 1.55 : Dwell-rise-dwell

According to Type of Constraint of the Follower

Pre-loaded Spring Cam

For its proper working there should be contact between the cam and the follower throughout its working, and it is achieved by means of a pre-loaded spring as shown in Figures 1.48(a) and (b), etc.

Positive Drive Cam

In this case, the contact between the cam and the follower is maintained by providing a roller at the operating end of the follower. This roller operates in the groove provided in the cam. The follower cannot come out of the groove, as shown in Figures 1.52 to 1.54.

Gravity Drive Cam

In this type of cam, the lift or rise of the follower is achieved by the rising surface of the cam (Figure 1.48(c)) and the follower returns or falls due to force of gravity of the follower. Such type of cams cannot be relied upon due to their uncertain characteristics.

1.12.3 Classification of Followers

Followers may be classified in three different ways :

(a) Depending upon the type of motion, i.e. reciprocating or oscillating.

(b) Depending upon the axis of the motion, i.e. radial or offset.

(c) Depending upon the shape of their contacting end with the cam.

Those of followers falling under classification (a) and (b) have already been dealt with as indicated above. Followers of type (c) will be taken up now.

Depending upon the Shape of their Contacting End with the Cam

Under this classification followers may be divided into three types :

(a) Knife-edge Follower (Figure 1.55(a))

(b) Roller Follower (Figure 1.55(b))

(c) Flat or Mushroom Follower (Figure 1.56(c))

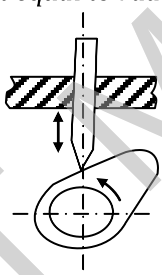
Knife-edge Follower

Knife-edge followers are generally, not used because of obvious high rate of wear at the knife edge. However, cam of any shape can be worked with it. During working, considerable side thrust exist between the follower and the guide.

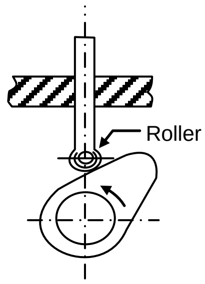
Roller Follower

In place of a knife edge, a roller is provided at the contacting end of the follower, hence, the name roller follower. Instead of sliding motion between the contacting surface of the follower and the cam, rolling motion takes place, with the result that rate of wear is greatly reduced. In roller followers also, as in knife edge follower, side thrust is exerted on the follower guide. Roller followers are extensively used in stationary gas and oil engines. They are also used in aircraft engines due to their limited wear at high cam velocity.

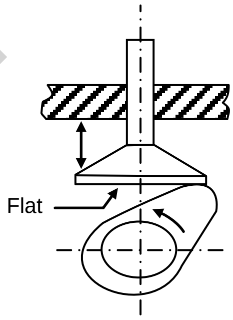
While working on concave surface of a cam the radius of the surface must be at least equal to radius of the roller.



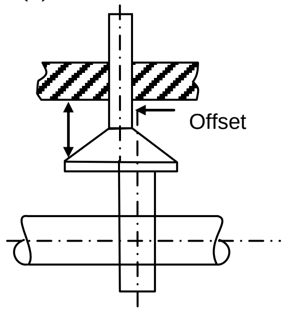
(a) Knife-edge Follower



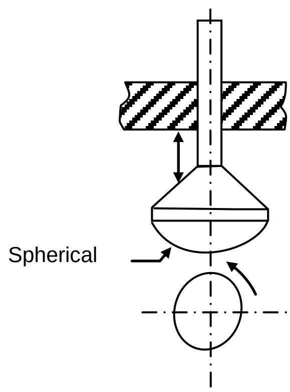
(b) Roller Follower



(c) Flat or Mushroom Follower



(i)



(d) : Spherical Follower

Figure 1.56 : Classification of Followers

Flat or Mushroom Follower

At the name implies the contacting end of the follower is flat as shown. In mushroom followers there is no side thrust on the guide except that due to friction at the contact of the cam and the follower. No doubt that there will be sliding motion between the contacting surface of the follower and the cam but the wear can be considerably reduced by off-setting the axis of the followers as shown in Figure 1.56(c)(i). The off-setting provided causes the follower to rotate about its own axis when the cam rotates.

Flat face follower is used where the space is limited. That is why it is used to operate valves of automobile engines. Where sufficient space is available as in stationary gas and oil engines, roller follower is used as mentioned above. The flat faced follower is generally preferred to the roller follower because of the compulsion of having to use small diameter of the pin in the roller of the roller follower.

In flat followers, high surface stresses are produced in the flat contacting surface. To minimise these stresses, spherical shape is given to the flat end, as shown in Figure 1.56(d). The curved faced or spherical faced followers are used in automobile engines.

With flat followers, it is obviously, essential that the working surface of the cam should be convex everywhere.

1.12.4 Terminology of Cam and Follower

The Cam Profile

The working contour of a cam which comes into contact with the follower to operate it, is known as the cam profile. In Figure 1.57, *A-B-C-D-A* is the cam profile or the working contour.

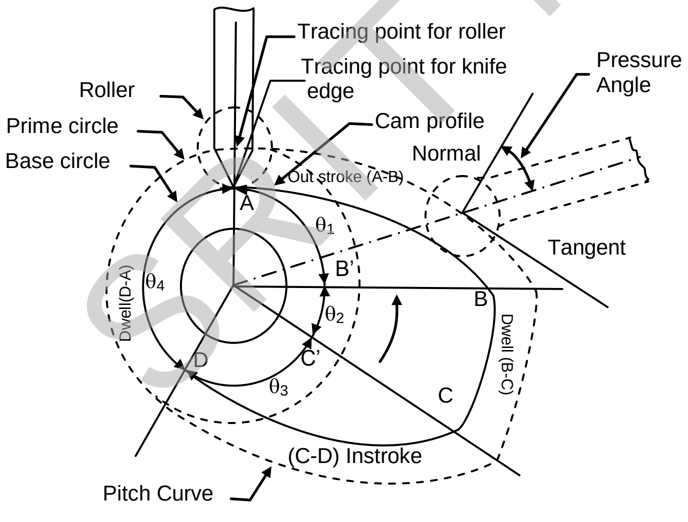


Figure 1.57 : Cam Profile

The Base Circle

The smallest circle, drawn from the centre of rotation of a cam, which forms part of the cam profile, is known as the *base circle* and its radius is called the least radius of the cam. A circle with centre *O* and of radius *OA* forms the base circle. Size of a cam depends upon the size of the base circle.

The Tracing Point

The point of the follower from which the profile of a cam is determined is called the *tracing point*. In case of a knife-edge follower, the knife edge itself is the tracing point. In roller follower, the centre of roller is the tracing point.

The Pitch Curve

The locus or path of the tracing point is known as the *pitch curve*. In knife-edge follower, the pitch curve itself will be the cam profile. In roller follower, the cam profile will be determined by subtracting the radius of the roller radially throughout the pitch curve.

The Prime Circle

The smallest circle drawn to the pitch curve from the centre of rotation of the cam is called as the *prime circle*. In knife-edge follower, the base circle and the prime circle are the same. In roller follower, the radius of the prime circle is the base circle radius plus the radius of the roller.

The Lift or Stroke

It is the maximum displacement of the follower from the base circle of the cam. It is also called as the *throw* of the cam. In Figure 1.57, distance *B′B* and *C′C* is the lift, for the roller follower.

The Angles of Ascent, Dwell, Descent and Action

Refer Figure 1.57, the angle covered by a cam for the follower to rise from its lowest position to the highest position is called the *angle of ascent* denoted as θ_1.

The angle covered by the cam during which the follower remains at rest at its highest position is called the *angle of dwell*, denoted by θ_2.

The angle covered by the cam, for the follower to fall from its highest position to the lowest position is called the *angle of descent* denoted as θ_3.

The total angle moved by the cam for the follower to return to its lowest position after the period of ascent, dwell and descent is called the *angle of action*. It is the sum of θ_1, θ_2 and θ_3.

The Pressure Angle

The angle included between the normal to the pitch curve at any point and the line of motion of the follower at the point, is known as the *pressure angle*. This angle represents the steepness of the cam profile and as such it is very important in cam design.

The Pitch Point

The point on the pitch curve having the maximum pressure angle is known as the *pitch point*.

The Cam Angle

It is the angle of rotation of the cam for a certain displacement of the follower.

1.13 MECHAICAL ADVANTAGE

The mechanical advantage of a mechanical is the ratio of the output force or torque at any instant to the input force or torque.

If friction and inertia forces are ignored.

Power input = Power output

If T_2 be input torque,

ω_2 be input angular velocity,

T_4 be output torque, and

ω_4 be output angular velocity.

$$T_2\ \omega_2 = T_4\ \omega_4$$

or $$\frac{T_4}{T_2} = \frac{\omega_2}{\omega_4}$$

$$\text{Mechanical Advantage } = \frac{T_4}{T_2} = \frac{\omega_2}{\omega_4}$$

Thus, mechanical advantage is the reciprocal of the velocity ratio.

1.14 SUMMARY

The kinematics of machine deals with analysis and synthesis of mechanisms. In the analysis, the kinematic quantities, i.e. displacement, velocity and acceleration of a point can be determined.

The velocity is defined as the time rate of change of displacement whereas acceleration is defined as time rate of change of velocity.

You have studied in this unit that the link is a basic element of a mechanism. For a planar mechanism, generally, minimum four links are required with four hinge joints with one link fixed. The motion of links takes place in plane. In a machine, at least one mechanism is used.

In this unit, you have studied that the degree of freedom depends on the constraints imposed on a moving body. The bodies which have constrained motion have practical utility. In order to form a closed kinematic chain, links are connected with each other. These connections are known as kinematic pairs. There are different types of mechanisms which use different types of kinematic pairs. Their major classification is based on nature of contact. Those having area contact are lower pairs and those having point contact or line contact are higher pairs. The pairs which provide constrained notion are used in kinematic chain. The mechanisms require links with multiple connections. A kinematic chain which consists of only binary links is termed a simple chain with hinges or joints. There are some special purpose mechanisms which have been discussed in this unit.

A four bar kinematic chain may have all the four kinematic pairs as revolute pairs, three revolute pairs and one prismatic pair or two revolute pairs and two prismatic pairs. These kinematic chains provide different mechanisms. Some of them are of common usage. The mechanisms obtained from kinematic chain using four revolute pairs depend on relative lengths of different links. The pantograph is used as a copying mechanism to enlarge the figure or shorten it. There are mechanisms for drawing exact straight line and approximate straight line. The steering gear mechanisms like Devi’s steering gear and Ackermann steering gear have been explained. Devi’s steering gear fulfils correct steering gear criteria whereas Ackermann steering gear does not fulfil correct steering gear criteria but it is being used universally because of its simplicity in construction and its cheaper maintenance. Hook’s coupling is used to connect non-aligned shafts which have angular misalignment. The speed of output shaft has variable speed even if speed of the input shaft is constant which is a drawback of Hook’s joint. The variation in speed depends on angle between the input and output shafts. The terminology and classification of the cam also has been explained in this unit.

1.15 KEY WORDS

Cam : It is a reciprocating, rotating or oscillating machine part or member which imparts motion to other member.

Follower : It is a reciprocating or oscillating member which follows motion of cam.

Machine	:	A machine is a mechanism or a collection of mechanisms which transmits force from the source of power to the resistance to be overcome, and thus performs useful mechanical work.
Mechanism	:	A mechanism is a combination of rigid or restraining bodies so shaped and connected that they move upon each other with definite relative motion. slider-crank mechanism used in internal combustion engine or reciprocating air compressor is the simplest example.
Kinematic Pair	:	A pair is a joint of two elements that permits relative motion. The relative motion between the elements of links that form a pair is required to be completely constrained or successfully constrained.
Kinematic Link or Element	:	Kinematic link is a resistant body or an assembly of resistant bodies which go to make a part or parts of a machine connecting other parts which have motion relative to it.
Resistant Body	:	A body is said to be a resistant body if it is capable of transmitting the required forces with negligible deformation.
Completely Constrained Motion	:	When the motion between a pair is limited to a definite direction, irrespective of direction of force applied and only one independent variable is required to define motion. Such a motion is said to be a completely constrained motion.
Incompletely Constrained Motion	:	When a connection between the elements, forming a pair, is such that the constrained motion is not completed by itself, but by some other means. Such a motion is said to be a successfully constrained motion.
Kinematic Chain	:	A kinematic chain is a combination of kinematic pairs, joined in such a way, that each link forms a part of two pairs and the relative motion between the links or elements is completely constrained.
Kinematic Inversions	:	In a mechanism, one of the links in a kinematic chain is fixed. One may obtain different mechanisms by fixing in different links in a kinematic chain. This method of obtaining different mechanisms is known as kinematic inversions.
Crank	:	The link which makes complete rotation.
Rocker	:	The link oscillates between two extreme positions.
Coupler	:	It transfers motion from input link to output link or vice-versa.

1.16 ANSWERS TO SAQs

Please refer the preceding text for all the Answers to SAQs.