UNIT 4 FLYWHEEL

Structure

4.1 INTRODUCTION

In practice, there are two following types of cases where reciprocating engine mechanism is used :

(a) An internal combustion engine or a steam engine which is used as a prime mover to drive generators, centrifugal pumps, etc.

(b) A punching machine which is driven by a prime mover like electric motor.

In both these cases either a variable torque is supplied where demand is a constant torque or demand is variable torque whereas constant torque is supplied. In both these cases there is mismatch between the supply and demand. This results in speed variation. In case of generators, speed variation results in change in frequency and variation in voltage. On the other hand, punching machine requires energy at small interval only when punching is done. To supply such large energy at the time of punching, motor of high power shall be required. At the same time, there will be large variation in speed. To smoothen these variations in torque, flywheel is used which works as a energy storage. This results in usage of low power motor in punching machine.

Objectives

After studying this unit, you should be able to

- explain the method of drawing turning moment diagram for a prime mover,
- determine the fluctuation of energy in a cycle,
- determine the power of prime power, and
- determine mass moment of inertia of a flywheel and design it.

4.2 DYNAMICALLY EQUIVALENT SYSTEM

The slider-crank mechanism is one of the most commonly used mechanism. It is used in prime movers, reciprocating compressors, punching machine, press, etc. The reciprocating mass comprises mass of the piston and part of the mass of the connecting rod. One end of the connecting rod reciprocates with piston and other end rotates with crank. We want to replace this link by a mass less link which is dynamically equivalent by having two point masses m_1, at the piston end and m_2 at the crank end.

In a general case, we can think of a rigid link of any shape as shown in Figure 4.1. Let this be subjected to a system of forces whose resultant is say '*F*' generating a couple *Fe* about centre of gravity *G*. This force produces linear acceleration '*a*' $\left(= \frac{F}{m}\right)$ and angular acceleration 'α' $\left(= \frac{F\,e}{I}\right)$, where *I* is mass moment of inertia about a perpendicular axis through *G*.

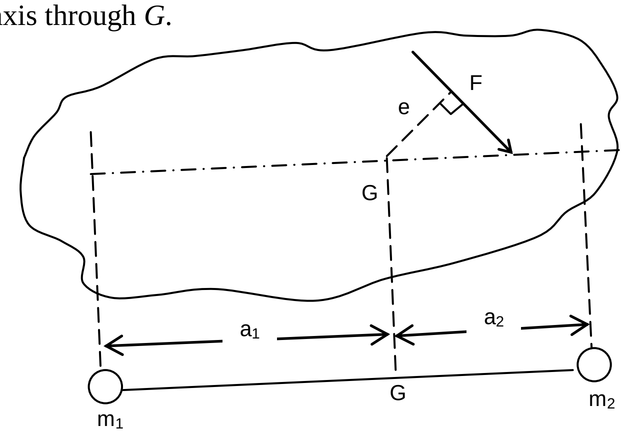


Figure 4.1 : Dynamically Equivalent System

For the massless link having point masses m_1 and m_2 to be dynamically equivalent, it should generate same accelerations '*a*' and 'α' due to the action of same force '*F*'.

Let a_1 and a_2 be the distances of the point masses m_1 and m_2 from centre of gravity *G*, respectively.

For a dynamically equivalent system having accelerations '*a*' and 'α'.

(a) Total mass should be same, i.e.

$$m_1 + m_2 = m \qquad \ldots (4.1)$$

(b) Position of centre of gravity should remain same, i.e.

$$m_1\, a_1 = m_2\, a_2 \qquad \ldots (4.2)$$

(c) Mass moment of inertia should be same, i.e.

$$m_1\, a_1^2 + m_2\, a_2^2 = I \qquad \ldots (4.3)$$

These three Eqs. (4.1) to (4.3) should be satisfied for complete dynamical equivalence. There are four unknowns (m_1, m_2, a_1 and a_2) and the three equations. Therefore, one of these four variables can be arbitrarily assumed and other three can be determined to provide a unique solution.

For slider-crank mechanism, it will be convenient to have mass m_1 with the piston and mass m_2 at the crank end and thus a_1 and a_2 are selected before hand. But in that case all the three equations cannot be satisfied.

From Eqs. (4.1) to (4.3),

$$m_1 = \frac{a_2}{(a_1 + a_2)}\, m \quad \text{and} \quad m_2 = \frac{a_1\, m}{(a_1 + a_2)}$$

and mass moment of inertia = $m\, a_1\, a_2$.

If masses m_1 and m_2 are located at above mentioned positions the Eq. (4.3) shall not be satisfied. The change in moment of inertia will be

$$= I - m\, a_1\, a_2$$

$$= m\, k^2 - m\, a_1\, a_2 = m\,(k^2 - a_1\, a_2)$$

The correction couple has to be determined and it will be $m\,(k^2 - a_1\, a_2)\,\alpha$. This couple can be thought of applied by the two forces F_C as shown in Figure 4.2.

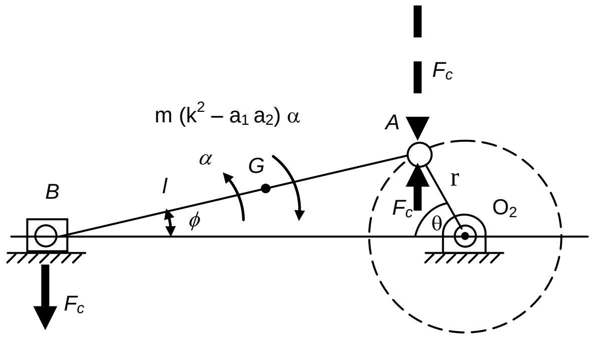


Figure 4.2 : Slider Crank Mechanism

Therefore, $$F_C\; l \cos\phi = m\,(k^2 - a_1\, a_2)\,\alpha$$

or, $$F_C = \frac{m\,(k^2 - a_1\, a_2)\,\alpha}{l \cos\phi} \qquad \ldots (4.4)$$

The correction in the turning moment will be equal to the moment of force F_C shown by the dashed line about the crank centre.

$$\text{Correction couple} = -\,F_C\; r\cos\theta = -\,\frac{m\,(k^2 - a_1\, a_2)\,\alpha}{l\cos\phi}\; r\cos\theta$$

Let $$\frac{r}{l} = \lambda$$

But $$\cos\phi = \left(1 - \frac{r^2}{l^2}\sin^2\theta\right)^{\frac{1}{2}}$$

$$= (1 - \lambda^2\sin^2\theta)^{\frac{1}{2}} = 1 - \frac{\lambda^2}{2}\sin^2\theta - \frac{\lambda^4}{8}\sin^4\theta + \ldots$$

Differentiating it w.r.t. 't' and dividing it by $\sin\phi = \lambda\sin\theta$

$$\frac{d\phi}{dt} = \omega\left(\lambda\cos\theta + \frac{\lambda^3}{2}\sin^2\theta\cos\theta + \ldots\right)$$

Differentiating again w.r.t. 't' and assuming 'ω' constant

$$\frac{d^2\,'\phi'}{dt^2} = \lambda\,(-\,\omega^2\,(\sin\theta))$$

(By approximation neglecting higher terms)

or, $$\frac{d^2\,'\phi'}{dt^2} = -\,\omega^2\,\frac{l}{r}\sin\theta = '\alpha'$$

Substituting for α

$$\text{Correction couple } 'M_c' = +\,\frac{m\,(k^2 - a_1\, a_2)}{l\cos\phi}\,\frac{\omega^2\, r^2}{2l}\sin 2\theta \qquad \ldots (11.5)$$

SAQ 1

What is the advantage of determining dynamically equivalent link for connecting rod?

4.3 TURNING MOMENT DIAGRAM

Figure 4.3 shows a layout of a horizontal engine.

Let p = effective gas pressure on the piston in N/m^2,

A = area of the piston in m^2,

m_{rec} = mass of reciprocating parts, i.e. mass of the piston gudgeon pin and part of mass of connecting rod 'm_1',

Q = thrust force on the connecting rod in N,

ω = angular velocity of the crank, and

M = Turning moment on the crank.

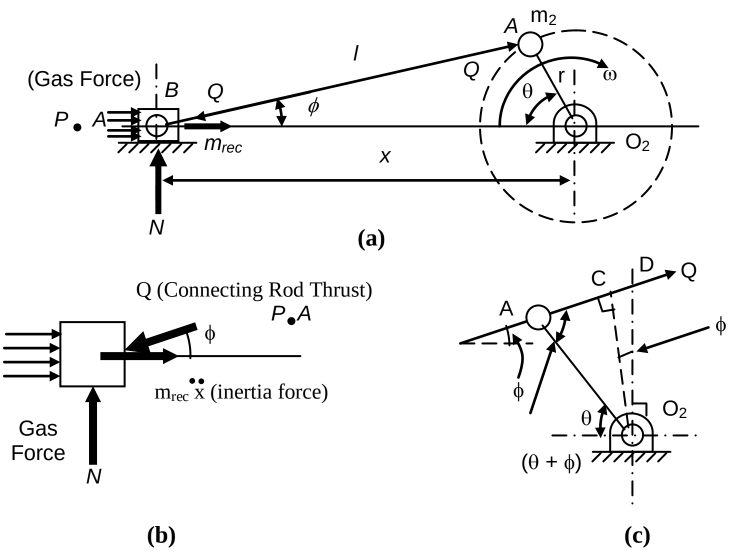


Figure 4.3 : Turning Moment Diagram

$$Q\cos\phi = pA + m_{rec}\,\ddot{x}$$

$$= pA - m_{rec}\,r\omega^2\left(\cos\theta + \frac{r}{l}\cos 2\theta\right)$$

or,
$$Q = \frac{pA - m_{rec}\,r\omega^2\left(\cos\theta + \frac{r}{l}\cos 2\theta\right)}{\cos\phi}$$

$$\therefore\quad M = Q\,r\sin(\theta+\phi) = \left\{pA - m_{rec}\,r\omega^2\left(\cos\theta + \frac{r}{l}\cos 2\theta\right)\right\}\frac{r\sin(\theta+\phi)}{\cos\phi}$$

$$= r\left\{pA - m_{rec}\,r\omega^2\left(\cos\theta + \frac{r}{l}\cos 2\theta\right)\right\}\frac{(\sin\theta\cos\phi + \cos\theta\sin\phi)}{\cos\phi}$$

$$= r\left\{pA - m_{rec}\,r\omega^2\left(\cos\theta + \frac{r}{l}\cos 2\theta\right)\right\}(\sin\theta + \cos\theta\tan\phi) \qquad \ldots (4.6)$$

or,
$$M = \left\{pA - m_{rec}\,r\omega^2\left(\cos\theta + \frac{r}{l}\cos 2\theta\right)\right\}O_2D \qquad \ldots (4.7)$$

In case of a vertical engine.

$$M = r\left\{pA - m_{rec}\, r\omega^2\left(\cos\theta + \frac{r}{l}\cos 2\theta\right) + m_{rec}\, g\right\}(\sin\theta + \cos\theta\tan\phi) \quad \ldots (4.8)$$

$$= \left\{pA - m_{rec}\, r\omega^2\left(\cos\theta + \frac{r}{l}\cos 2\theta\right) + m_{rec}\, g\right\} O_2 D \quad \ldots (4.9)$$

Considering the correction couple also, the actual turning moment is

$$M_t = M + M_C$$

4.3.1 Turning Moment Diagram of a Single Cylinder 4-stroke IC Engine

If the effect of correction couple is ignored, the approximate turning moment

$$M = (\text{Gas force} + \text{Inertia force})\; O_2 D$$

The diagram which is plotted for 'M' against crank angle 'θ' is called turning moment diagram. This diagram can be plotted progressively as explained below :

(a) There are two forces, i.e. gas force and inertia force.

Gas force = $p \times$ Piston area

where p is the gas pressure.

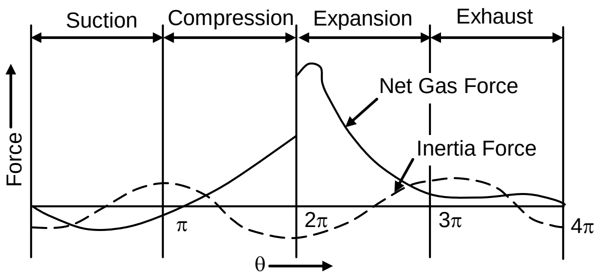


Figure 4.4(a)

The variation in the gas force will be due to the change in pressure. The gas force and inertia force have been plotted in Figure 4.4(a) for all the four strokes.

(b) The net force is the resultant of gas force and inertia force. It can be plotted in reference to θ as shown in Figure 4.4(b).

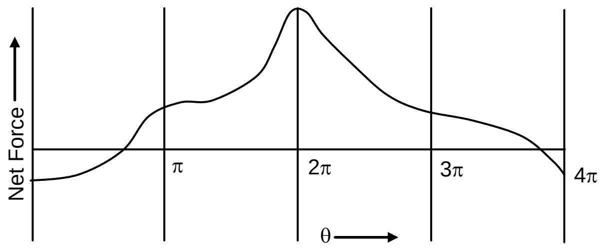


Figure 4.4(b)

(c) The value of O_2D is given by

$$O_2D = r\,(\sin\theta + \cos\theta\tan\phi)$$

For various values of θ, O_2D can be determined and then plotted. The plot of this is shown in Figure 4.4(c).

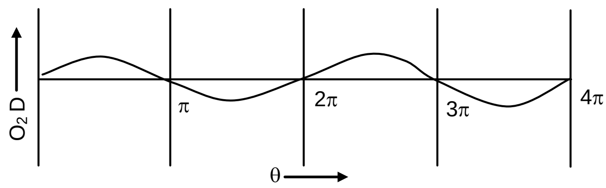


Figure 4.4(c)

(d) The approximate turning moment '*M*' = Net force × O_2D. The plot of '*M*' Vs θ is shown in Figure 4.4(d).

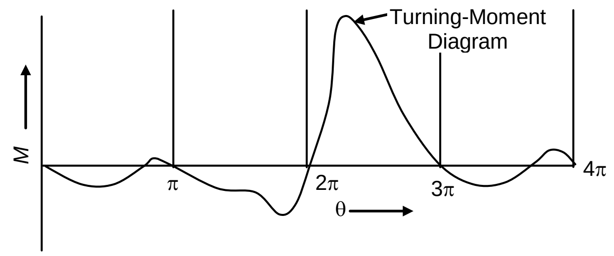


Figure 4.4(d)

The turning moment in the suction stroke and exhaust stroke is very small. In case of compression stroke and expansion stroke turning moment is higher. In compression stroke, energy is to be supplied and in expansion stroke, large amount of energy is available. By surveying the turning moment diagram, it is observed that the energy is supplied in three strokes and energy is available only in one stroke. Therefore, three strokes, i.e. suction stroke, compression, and exhaust stroke the engine is starving of energy and in expansion stroke it is harvesting energy. At the same time it is observed that there is large variation of turning moment during the cycle. The variation in the turning moment results in corresponding variation in speed of the crank.

SAQ 2

What do you mean by turning moment diagram?

4.3.2 Turning Moment Diagram of a Multicylinder 4-stroke IC Engine

In case of multi cylinder engine there will be more expansion strokes. For example, in the case of three cylinder engine, there will be three expansion strokes in each cycle. In case of 4 cylinder 4-strokes engine there will be four expansion strokes. Therefore, in multi cylinder engine there will be lesser variation in turning moment as compared to single cylinder engine and consequently there is expected to be less variation in speed. The turning moment diagram for a multi cylinder engine is expected to be as shown in Figure 4.5. Therefore the variation in the turning moment reduces with the increase in the number of cylinders.

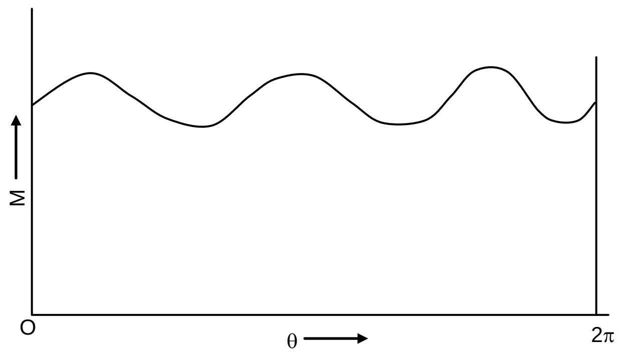
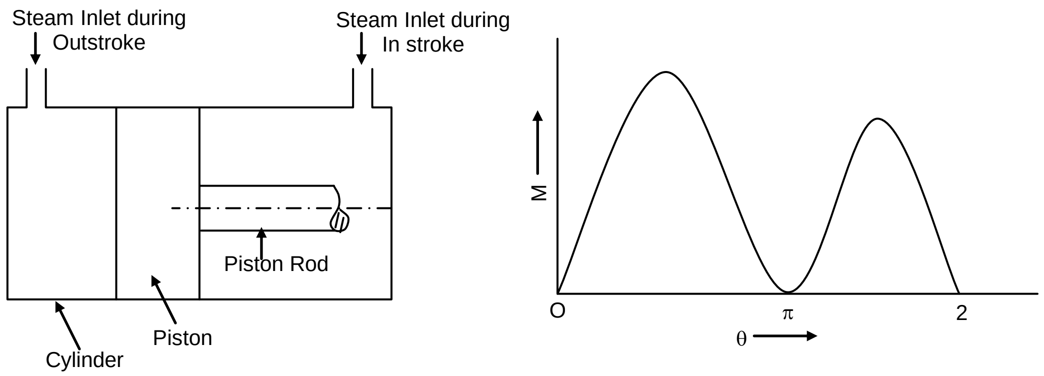


Figure 4.5 : Turning Moment Diagram

4.3.3 Turning Moment Diagram of a Single Cylinder Double Acting Steam Engine

The cylinder and piston arrangement of the steam engine is shown in Figure 4.6(a) and turning moment diagram is shown in Figure 4.6(b).



(a) **(b)**

Figure 4.6 : Turning Moment Diagram of a Single Cylinder Double Acting Steam Engine

For outstroke, force = steam pressure × area of the piston.

For instroke, force = steam pressure × (area of piston – area of piston rod).

During out stroke the area over which steam pressure acts is more as compared to in stroke where some of the area is occupied by the piston rod. Because of the difference in the available areas there is difference in the maximum turning moments in the two strokes. Steam pressure is nearly constant and variation in the turning moment is due to the value of O_2D and inertia force of the reciprocating masses. As compared to the single cylinder 4-stroke engine, the variation in turning moment is less in case of double acting steam engine.

SAQ 3

Why variation in the turning moment of single cylinder 4-stroke IC engine is more as compared to the multi cylinder IC engines?

4.4 FLUCTUATION OF ENERGY AND SPEED

As shown in Figures 4.4 to 4.6, the turning moment 'M' varies considerably whereas the resisting moment say 'M_R' which is due to the machine to be driven remains constant over a cycle for most of the cases. If we superimpose the resisting moment over the turning moment diagram, a situation shown in Figure 4.7 will arise. If M_R is equal to the average turning moment (M_{av}), energy available shall be equal to the energy required over a cycle. It can be observed that for some values of θ turning moment is more than M_R and for some values of θ turning moment is less than M_R.

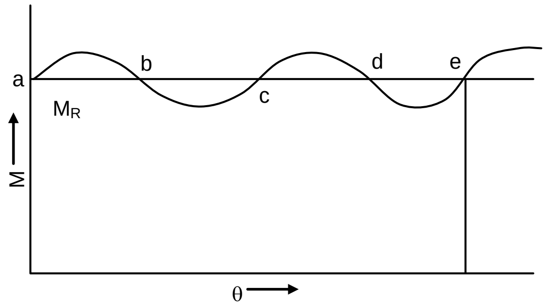


Figure 4.7 : Fluctuation of Energy and Speed

The energy output can be expressed mathematically as follows :

$$E = \int M \, d\,\theta$$

The average turning moment for the cycle is

$$M_{av} = \frac{E}{\text{Angle for cycle}}$$

The angle for the cycle is 2π for the two stroke engines and 4π for four strokes engines and in case of steam engines it is 2π.

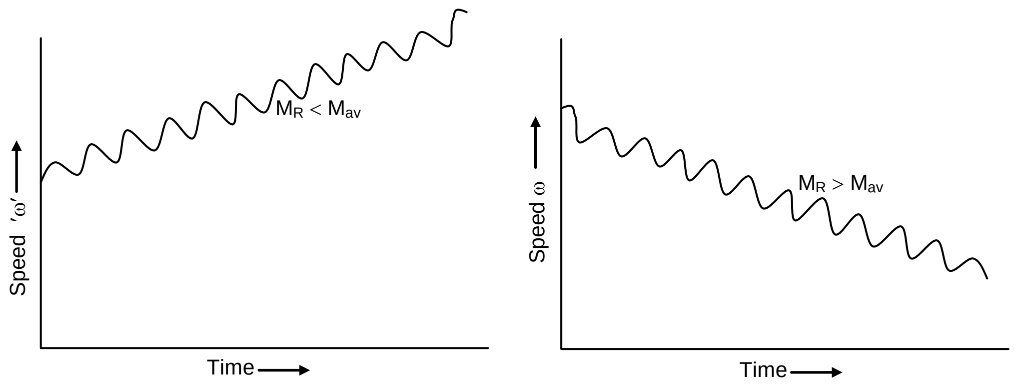
For a stable operation of the system

$$M_R = M_{av}$$

In the stable system, the mean speed remains constant but variation of speed will be there within the cycle. The speed remains same at the beginning and at the end of the cycle.

If $M_R < M_{av}$, the speed increases from cycle to cycle. The speed graph is shown in Figure 4.8(a).

If $M_R > M_{av}$, the speed decreases from the cycle to the cycle. The speed graph is shown in Figure 4.8(b).



(a) **(b)**

Figure 4.8 : Speed Graph

From Figure 4.7, we observe that $M_R = M_{av}$ at points *a*, *b*, *c*, *d* and *e*. Since $M > M_R$ from *a* to *b*, speed of the crank shaft will increase during this period. From *b* to *c* $M < M_R$ and speed will decrease. Similar situation will occur for *c* to *d* and *d* to *e*. At *e* the cycle is complete and the speed at *e* is same as that of *a*. The energy at all these points can be determined.

$$E_b = E_a + \int_{\theta a}^{\theta b} (M - M_R)\, d\theta$$

$$E_c = E_b + \int_{\theta b}^{\theta c} (M - M_R)\, d\theta$$

$$E_d = E_c + \int_{\theta c}^{\theta d} (M - M_R)\, d\theta$$

$$E_e = E_d + \int_{\theta d}^{\theta e} (M - M_R)\, d\theta = E_a$$

Out of all these energies so determined, we can find minimum and maximum energies, the difference in these energy levels shall give maximum fluctuation of energy $(\Delta E)_{max}$

$$(\Delta E)_{max} = E_{max} - E_{min}$$

The coefficient of fluctuation of energy is the ratio of maximum fluctuation of energy to the energy of cycle

$$\therefore \qquad k_e = \frac{(\Delta E)_{max}}{E} \qquad \text{. . . (4.10)}$$

The maximum energy level point shall have maximum speed and minimum energy level point shall have minimum speed. The coefficient of fluctuation of speed is defined as follows :

$$k_s = \frac{\omega_{max} - \omega_{min}}{\omega_{av}} = \frac{2\,(\omega_{max} - \omega_{min})}{(\omega_{max} + \omega_{min})} \qquad \text{. . . (4.11)}$$

SAQ 4

In which type of engine speed fluctuation will be maximum and why?

4.5 FLYWHEEL DESIGN

It has been discussed in the preceding section that fluctuation of energy results in fluctuation of the crank shaft speed which then results in fluctuation of the kinetic energy of the rotating parts. But the maximum permissible fluctuation in speed of the crank shaft is determined by the purpose for which the engine is to be used. Therefore, to keep the maximum fluctuation of speed within a specific limit for a given maximum fluctuation of energy, a flywheel is mounted on the crank shaft.

4.5.1 Mass Moment of Inertia of Flywheel for an IC Engine

The function of the flywheel is to store excess energy during period of harvestation and it supplies energy during period of starvation. Thereby, it reduces fluctuation in the speed within the cycle. Let ω_1 be the maximum angular speed and ω_2 be the minimum angular speed.

Let *I* be the mass moment of inertia of the flywheel.

Neglecting mass moment of inertia of the other rotating parts which is negligible in comparison to mass moment of inertia of the flywheel.

Maximum kinetic energy of flywheel

$$(\text{K.E.})_{max} = \frac{1}{2} I\ \omega_1^2$$

Minimum kinetic energy of flywheel

$$(\text{K.E.})_{min} = \frac{1}{2} I\ \omega_2^2$$

Change in K.E., i.e. $\Delta\ \text{K.E.} = \frac{1}{2} I\ (\omega_1^2 - \omega_2^2)$

Δ K.E. = fluctuation in energy, i.e. ΔE

$$\therefore \qquad \Delta E = \frac{1}{2} I\ (\omega_1^2 - \omega_2^2) \qquad \text{. . . (4.12)}$$

or,

$$\Delta E = \frac{1}{2} I\ (\omega_1 - \omega_2)\ (\omega_1 + \omega_2)$$

$$= \frac{1}{2} I\ (\omega_1 - \omega_2) \times 2\omega$$

where 'ω' is average speed given by

$$\omega = \frac{(\omega_1 + \omega_2)}{2}$$

or, $$\Delta E = \frac{1}{2} I \frac{(\omega_1 - \omega_2)}{w} \times \omega^2$$

or, $$\Delta E = I\ k_s\ \omega^2 \qquad \text{. . . (4.13)}$$

Energy fluctuation can be determined from the turning moment diagram. For selected value of k_s, and given value of speed ω, I can be determined.

Eq. (4.12) can also be written as follows :

$$\Delta E = \frac{1}{2} M\ k^2\ (\omega_1^2 - \omega_2^2)$$

where k is the radius of gyration and M is mass of the flywheel

or, $$\Delta E = \frac{1}{2} M\ \{(k\,\omega_1)^2 - (k\,\omega_2)^2\}$$

Let V_1 be the maximum tangential velocity at the radius of gyration and V_2 be the minimum tangential velocity at the radius of gyration

∴ $$V_1 = k\,\omega_1 \quad \text{and} \quad V_2 = k\,\omega_2$$

and $$\Delta E = \frac{1}{2} M\ (V_1^2 - V_2^2) \qquad \text{. . . (4.14)}$$

It can be observed from Eq. (4.13) that

(a) The flywheel will be heavy and of large size if Δ E is large. The value of k_s is limited by the practical considerations. Therefore, single cylinder 4-stroke engine shall require larger flywheel as compared to the multi-cylinder engine.

(b) For slow speed engine also the flywheel required is larger in size because of high value of I required.

(c) For high speed engines, the size of flywheel shall be considerably smaller because of lower value of I required.

(d) If system can tolerate considerably higher speed fluctuations, the size of flywheel will also be smaller for same value of Δ E.

Example 4.1

The turning moment diagram for a multi cylinder IC engine is drawn to the following scales

1 cm = 15° crank angle

1 cm = 3 k Nm

During one revolution of the crank the areas with reference to the mean torque line are 3.52, (–) 3.77, 3.62, (–) 4.35, 4.40 and (–) 3.42 cm². Determine mass moment of inertia to keep the fluctuation of mean speed within ± 2.5% with reference to mean speed. Engine speed is 200 rpm.

Solution

The turning moment diagram is shown in Figure 4.9.

The scales are

1 cm = 15° crank angle

1 cm = 3 k Nm

Therefore, $$1\ \text{cm}^2 = 3000 \times \frac{\pi}{180} \times 15 = 785\ \text{Nm}$$

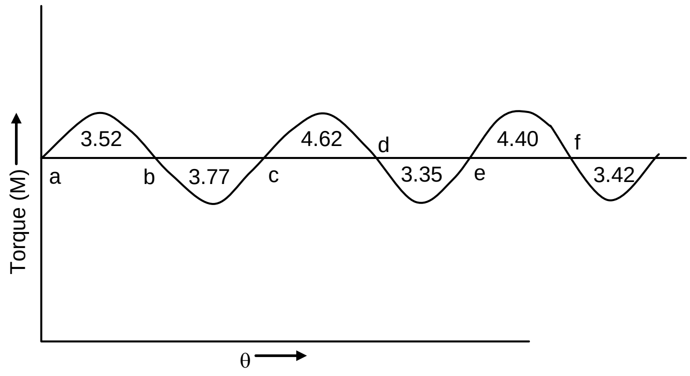


Figure 4.9 : Figure for Example 4.1

The overall speed fluctuation = 2 × 2.5%

∴ Coefficient of speed fluctuation ‘k_s’ = 0.05

Engine speed = 200 rpm

∴ $$\omega = \frac{2\pi \times 200}{60} = 20.93 \text{ r/s}$$

Let Energy level at *a* is ‘*E*’ cm^2

Energy level at *b* is $E_b = E + 3.52$

Energy level at *c* is $E_c = E_b - 3.77 = E + 3.52 - 3.77 = E - 0.25$

Energy level at *d* is $E_d = E_c + 3.62 = E - 0.25 + 3.62 = E + 3.37$

Energy level at *e* is $E_e = E_d - 4.35 = E + 3.37 - 4.35 = E - 0.98$

Energy level at *f* is $E_f = E_e + 4.40 = E - 0.98 + 4.40 = E + 3.42$

Energy level at *g* is $E_g = E_f - 3.42 = E + 3.42 - 3.42 = E$

Energy level at the end of cycle and at the beginning of the cycle should be same. By comparing the values of energies at various points, we get

Maximum energy is at ‘*b*’, i.e. $E_{\max} = E + 3.52$

Minimum energy is at ‘*e*’, i.e. $E_{\min} = E - 0.98$

Since, $$\Delta E = I\ k_s\ \omega^2$$

∴ $$3532.5 = I \times 0.05 \times (20.93)^2$$

or, $$I = \frac{3532.5}{0.05 \times (20.93)^2} = \frac{3532.5}{20.9} = 169 \text{ kg m}^2$$

Example 4.2

A single cylinder four-stroke petrol engine develops 18.4 kW power at a mean speed of 300 rpm. The work done during suction and exhaust strokes can be neglected. The work done by the gases during explosion strokes is three times the work done on the gases during the compression strokes and they can be represented by the triangles. Determine the mass of the flywheel to prevent a fluctuation of speed greater than 2 per cent from the mean speed. The flywheel diameter may be taken as 1.5 m.

Solution

Power developed ‘*P*’ = 18.4 kW

Mean speed ‘*N*’ = 300 rpm

Fluctuation of speed = ± 2%

Diameter of flywheel = 1.5 m

The turning moment diagram is shown in Figure 4.10.

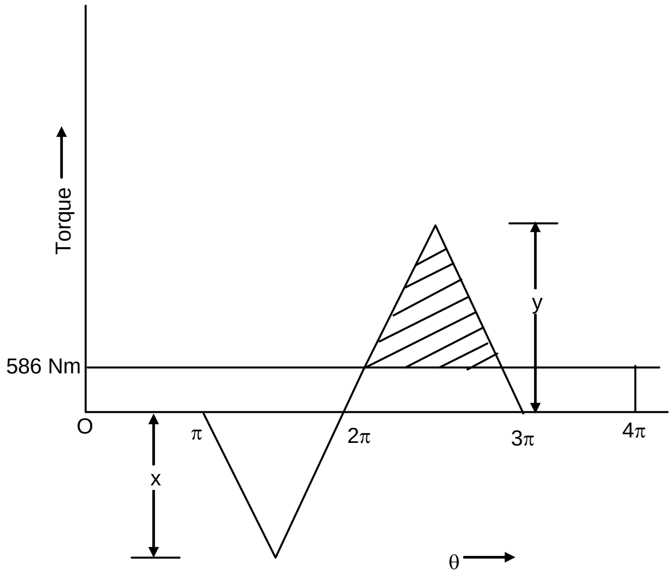


Figure 4.10 : Figure for Example 4.2

Let height of the triangle in compression stroke by 'x' and that in explosion stroke be 'y'. Since, work done in explosion stroke is three times to that in compression stroke.

$$\therefore \quad \frac{\pi y}{2} = 3\,\frac{\pi x}{2} \quad \text{or} \quad y = 3x$$

The net work done per second = 18.4 × 1000 Nm

Numbers of cycles per second $= \dfrac{300}{60 \times 2} = 2.5$

Therefore, work done per cycle $= \dfrac{18400}{2.5} = 7360$ Nm

From turning moment diagram, work done per cycle $= \dfrac{\pi y}{2} - \dfrac{\pi x}{2} = \dfrac{\pi}{2}(y - x)$

$$\therefore \quad \frac{\pi}{2}(y - x) = 7360$$

$$\text{or,} \quad y - x = \frac{7360 \times 2}{\pi} = 4687.9$$

Substituting for y in the above expression.

$$3x - x = 4687.9 \quad \text{or} \quad x = \frac{4687.9}{2}$$

$$\text{or,} \quad x = 2343.95 \text{ kN}$$

$$\therefore \quad y = 3x = 7031.85 \text{ Nm}$$

Average net torque $= \dfrac{7360}{4\pi} = 586$ Nm

The fluctuation of energy will be provided by the shaded area shown in Figure 4.10. From the theory of similar triangles

$$\frac{\text{Base of shaded area}}{\pi} = \frac{7031.85 - 586}{7031.85}$$

$$\therefore \quad \text{Base of shaded area } = \frac{\pi \times 6445.85}{7031.85} = 2.878$$

Therefore, fluctuation of energy $\Delta E = \dfrac{2.878 \times 6445.85}{2} = 9276.63$ Nm

$$\Delta E = I\ k_s\ \omega^2$$

$$k_s = 2 \times 2\% = 4\% = 0.04$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 300}{60} = 10\pi = 31.4 \text{ r/s}$$

$$\therefore \quad 9276.63 = I \times 0.04 \times (31.4)^2$$

or, $$I = \frac{9276.63}{0.04 \times (31.4)^2} = 235.22 \text{ kg m}^2$$

Assuming that the entire mass is concentrated at the rim

$$I = M R^2$$

where R is radius of the rim.

$$R = \frac{1.5}{2} = 0.75 \text{ m}$$

$$\therefore \quad 235.22 = M\,(0.75)^2$$

$$\therefore \quad M = \frac{235.22}{(0.75)^2} = 418.17 \text{ kg}$$

Example 4.3

The torque developed by an IC engine is given by

$$M = (1000 + 300 \sin 2\theta - 500 \cos 2\theta) \text{ Nm}$$

where θ is the angle turned by the crank from inner dead centre.

The engine speed is 300 rpm. The mass of the flywheel is 200 kg and radius of gyration is 400 mm. Determine

(a) power developed by the engine,

(b) percentage fluctuation of speed with reference to the mean speed,

(c) angular acceleration of the flywheel when the crank has rotated 60° from the inner dead centre, and

(d) maximum angular acceleration and the retardation of the flywheel.

Solution

Since the expression of the torque is a function of ‘2θ’, therefore, the angle for the cycle will be ‘π’ because the torque will be repeated in every ‘π’ angle.

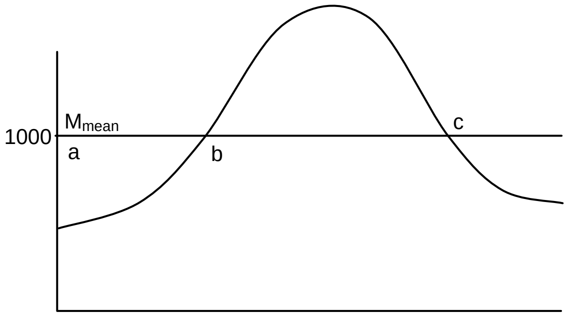


Figure 4.11 : Figure for Example 4.3

(a) The work done per cycle $= \int_0^{\pi} M$

$$= \int_0^{\pi} (1000 + 300 \sin 2\theta - 500 \cos 2\theta)\, d\theta$$

$$= 1000\,\theta - \frac{300}{2} \cos 2\theta - \frac{500}{2} \sin 2\theta \Big|_0^{\pi}$$

$$= 1000\,\pi \text{ Nm}$$

Number of cycles per second $= \dfrac{300}{60} \times 2 = 10$

$\therefore$ Word done per second = 1000 $\pi \times 10$

or, Power developed = 10000 π = 31400 = 31.4 kW.

(b) In order to determine fluctuation of the energy we should first determine location of points b and c as shown in Figure 4.11.

$$\text{Mean torque } (M_{\text{mean}}) = \frac{\text{Work done per cycle}}{\text{Angle for cycle}}$$

$$= \frac{1000\,\pi}{\pi} = 1000 \text{ Nm}$$

$\therefore$ $M - M_{\text{mean}} = 0$

or, $= 1000 + 300 \sin 2\theta - 500 \cos 2\theta - 1000 = 0$

or, $300 \sin 2\theta - 500 \cos 2\theta = 0$

or, $\tan 2\theta = \dfrac{500}{300} = \dfrac{5}{3}$

or, $2\theta = 59 \quad \text{or} \quad 239^{o} \quad \text{or} \quad \theta = 29.5^{o} \quad \text{or} \quad 119.5^{o}$

The fluctuation of energy

$$\Delta E = \int_{29.5}^{119.5} (1000 + 300 \sin 2\theta - 500 \cos \theta - 1000)\, d\theta$$

$$= 583 \text{ Nm}$$

Since, $\Delta E = I\, k_s\, \omega^2$

$$I = m\,k^2 = 200\,(0.4)^2 = 32 \text{ kg m}^2$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi\, 300}{60} = 10\pi = 31.4 \text{ r/s}$$

$\therefore$ $583 = 32\, k_s \times (31.4)^2$

or, $k_s = \dfrac{583}{32 \times (31.4)^2} = 0.018$

$\therefore$ % fluctuation of speed with reference to mean speed

$$= \frac{0.018 \times 100}{2} = 0.9\,.$$

(c) Acceleration is produced by the excess torque.

Excess torque = 300 sin 2θ – 400 cos 2θ

when $\theta = 60^o$

$$\text{Excess torque at } \theta = 60^o = 300 \sin 120^o - 500 \cos 120^o$$

$$= 259.8 + 250 = 509.8$$

$I\,\alpha = 509.8$

or, Angular acceleration $\alpha = \frac{509.8}{32} = 15.93$

or, $\alpha = 15.93$ r/s

(d) Maximum acceleration shall occur where torque is maximum and minimum acceleration will occur where torque is minimum.

$$\therefore \quad \frac{dM}{d\theta} = 2 \times 300 \cos 2\theta + 2 \times 500 \sin 2\theta = 0$$

or, $\tan 2\theta = -0.6$

∴ $2\theta = 149.04^o, 329.04^o$ or $\theta = 74.52^o, 164.52^o$

when $\theta = 74.52^o$; $M = 1583.1$ Nm

when $\theta = 164.52^o$; $M = 416.9$ Nm

For maximum acceleration $I\,\alpha = 1583.1 - 1000$

$$\therefore \quad \alpha_{max} = \frac{583.1}{32} = 18.22 \text{ r/s}^2$$

For minimum acceleration $I\,\alpha = 416.9 - 1000$

$$\therefore \quad \alpha_{min} = \frac{-583.1}{32} = -18.22 \text{ r/s}^2$$

Example 4.4

A three cylinder two-stroke engine has its cranks 120^o apart. The speed of the engine is 600 rpm. The turning moment diagram for each cylinder can be represented by a triangle for one expansion stroke with a maximum value of one stroke with a maximum value of 600 Nm at 60^o from the top dead centre. The turning moment in other stroke is zero for all the cylinders. Determine :

(a) the power developed by the engine,

(b) the coefficient of fluctuation of speed with a flywheel having mass 10 kg and radius of gyration equal to 0.5 m,

(c) the coefficient of fluctuation of energy, and

(d) the maximum angular acceleration of the flywheel.

Solution

The turning moment diagram of the engine is shown in Figure 4.12. The resultant diagram is also shown which has shaded areas.

(a) Work done per cycle

$$= 3\left(600 \times \frac{\pi}{2}\right) = 900\,\pi$$

Mean torque $M_{av} = \frac{900\pi}{2\pi} = 450$ Nm

$\therefore$ Power developed $'P' = M_{av} \times \frac{2\pi N}{60}$

$$= 450 \times \frac{2\pi\ 600}{60}$$

$$= 4500 \times 2\pi$$

$$= 9000\pi$$

= 28260 Watt

or, $P = 28.260$ kW

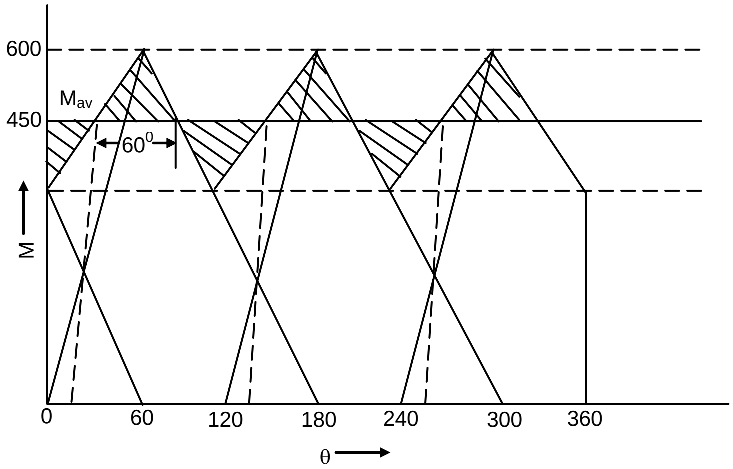


Figure 4.12 : Figure for Example 4.4

(b) Mass moment of inertia of the flywheel

$'I' = m\,k^2$

$= 10 \times (0.5)^2$

$= 2.5$ kg m^2

The fluctuation of energy is given by the shaded area above the mean line

$$\Delta E = \frac{(600 - 450)}{2} \times \frac{60}{180} \times \pi = \frac{150}{6} \times \pi = 78.5 \text{ Nm}$$

Speed, $w = \frac{2\pi N}{60} \times \frac{2\pi\ 600}{60} = 20\pi = 62.8$ r/s

$$\Delta E = I\ k_s\ w^2$$

or, $78.54 = 2.5\ k_s\ (62.8)^2$

or, $k_s = \frac{78.5}{2.5 \times (62.8)^2} = \frac{78.5}{2.5 \times 3943.84}$

Coefficient of fluctuation = 0.008.

(c) Coefficient of fluctuation of energy $= \frac{\Delta E}{E}$

$\Delta E = 78.5$ and $E = 900\pi$

Therefore, coefficient of fluctuation of energy $= \frac{78.5}{900\pi} = 0.028$

(d) Maximum excess torque = 600 – 450 = 150 Nm

Let angular acceleration be 'α'.

$\therefore$ $I\,\alpha_{max} = 150$

$$\alpha_{max} = \frac{150}{I} = \frac{150}{2.5} = 60 \text{ r/s}^2$$

4.5.2 Mass Moment of Inertia of Flywheel for a Punching Press

In this case torque supplied is constant because these machines are driven by the electric motor but the demand torque, i.e. resisting torque varies during cycle. The example of them are punching press, shearing machine, etc.

The schematic diagram of punching press is shown in Figure 4.13. In place of slider in slider crank mechanism, punching tool is used. Since motor is used to drive this press, the torque supplied shall be constant. On the other hand, high resisting torque will act when punching operation is done, i.e. from $\theta = \theta_1$ to θ_2. After this operation the resisting torque will be almost zero. Unless a flywheel is used, the speed of the crank shaft will be very high when resisting torque is very small and substantial decrease in speed shall take place when punching operation is done. If flywheel is provided, the excess energy shall be absorbed in the flywheel and it will be available when punching operation is being done where energy is deficient. It will result in reduction of the power of motor required if a suitable flywheel is used.

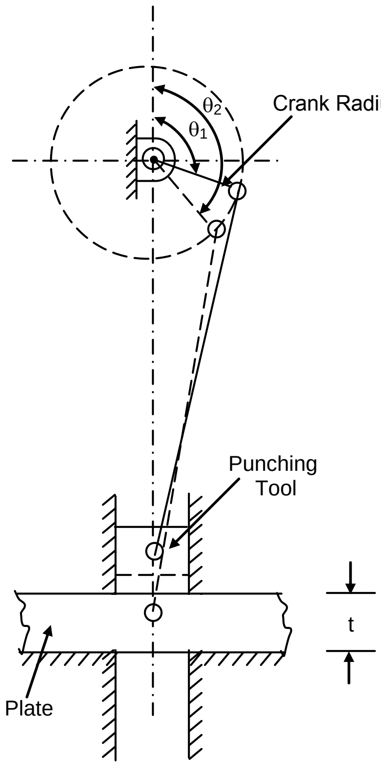


Figure 4.13 : Punching Press

Let E be the energy required for punching one hole. For a stable operation, the energy supplied to the crank for one revolution should also be equal to E.

The fluctuation of energy 'ΔE' = Energy required for one punch – Energy supplied during punching

Energy E = Work done in punching hole

$$= \left(\frac{1}{2}\right) \{\text{Maximum force } (P)\} \times \text{Thickness of plate}$$

Here, P = Sheared area × Shear strength

Energy supplied during punching $= \frac{E\,(\theta_2 - \theta_1)}{2\pi}$

$$\therefore \qquad \Delta E = E - \frac{E\,(\theta_2 - \theta_1)}{2\pi} = E\left\{1 - \frac{\theta_2 - \theta_1}{2\pi}\right\}$$

Angles θ_1 and θ_2 should be in radians.

Let *t* be the thickness of the plate in which holes are to be punched.

s be the length of the stroke.

r be the length of the crank.

l be the length of the connecting rod.

θ_1 and θ_2 can be determined geometrically if *r*, *l* and *t* are known.

The rough estimate can also be made as follows :

$$\frac{\theta_2 - \theta_1}{2\pi} \approx \frac{t}{2s} = \frac{t}{4r}$$

$$\therefore \qquad \Delta E = E\left\{1 - \frac{(\theta_2 - \theta_1)}{2\pi}\right\} = E\left(1 - \frac{t}{4r}\right) = I\,k_s\,\omega^2 \qquad \ldots (4.15)$$

The mass moment of inertia of the flywheel for a given value of coefficient of fluctuation of speed can be determined. In order to reduce the size of flywheel it will be better to mount the flywheel on a shaft having higher value of ‘ω’.

SAQ 5

Between the motor shaft and shaft carrying the punching tool there is heavy speed reduction. On which shaft flywheel should be mounted in order to have smaller flywheel.

4.5.3 Design of Flywheel

The flywheel has a heavy rim which is connected to the hub by several arms. The mass moment of inertia is largely contributed by the rim. From experience, it is known that 85 to 90% of the total mass can be assumed to be estimated in the rim. 15 to 10 percent of the mass is in the hub and arms.

The mean rim radius, R_m, depends on allowable tensile strength, σ_a, mass density, ρ, and maximum angular velocity, ω_{max}, and it is given by

$$R_m = \frac{1}{\omega_{max}} \times \left(\frac{\sigma_a}{\rho}\right)^{\frac{1}{2}}$$

The cross-section of the rim is rectangular having more width than the thickness. The width may be about 2.5 times of thickness. The material used is generally cast iron.

Example 4.5

The resisting torque on the crank of a riveting machine is 200 Nm for first 90°, from 90° to 135° is 1600 Nm then it drops linearly to 200 Nm upto 180° and remains the same upto 360°. The duration of cycle is 2 sec. The motor driving the machine, however, has a speed of 1450 rpm and it delivers constant torque. The crank shaft of the machine is geared to the motor shaft. The speed fluctuation is limited to ± 2% of mean speed. Determine :

(a) power of the motor, and

(b) moment of inertia of the flywheel mounted on the motor shaft.

Solution

In this problem, torque supplied is constant and demand torque is fluctuating. The demand torque is shown in Figure 4.14.

(a) The energy required per cycle

$$E = 200\left(\frac{\pi}{2} + \pi\right) + 1600\left(\frac{3\pi}{4} - \frac{\pi}{2}\right) + (1600 - 200)\,\frac{\pi}{4} \times \frac{1}{2} + 200 \times \frac{\pi}{4}$$

$$= 200 \times \frac{3}{2}\,\pi + 1600 \times \frac{\pi}{4} + 1400 \times \frac{\pi}{8} + 200\,\frac{\pi}{4}$$

$$= \left(300 + 400 + \frac{700}{4} + 50\right)\pi = (750 + 175)\,\pi = 925\,\pi$$

$$= 2904.5 \text{ Nm}$$

The duration of the cycle is 2 seconds

$\therefore$ Energy required per second $= \dfrac{2904.5}{2} = 1452.25$

$\therefore$ Power of motor required, $p = 1.452$ kW.

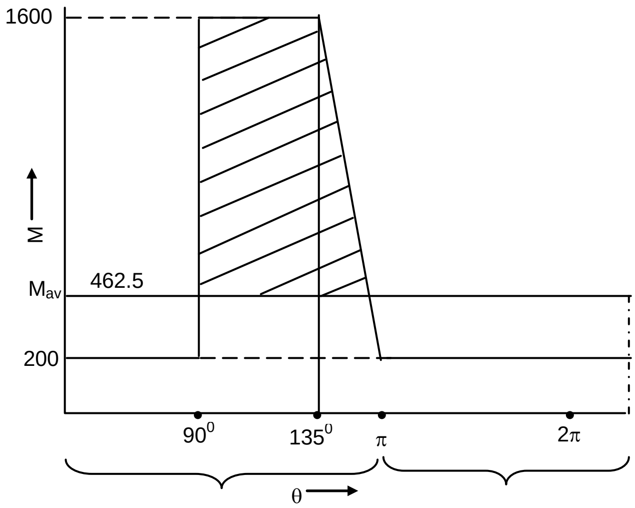


Figure 4.14 : Figure for Example 4.5

(b) The average torque $M_{av} = \dfrac{E}{2\pi} = \dfrac{2904.5}{2\pi}$

or, $M_{av} = 462.5$ Nm

The shaded portion is fluctuation of energy. Therefore,

$$\Delta E = (1600 - 462.5)\left(\frac{3\pi}{4} - \frac{\pi}{2}\right) + \frac{(1600 - 462.5)}{2}\left\{\frac{\left(\frac{\pi}{4}\right)(1600 - 462.5)}{(1600 - 200)}\right\}$$

$$= 1137.5 \times \frac{\pi}{4} + \frac{1137.5^2}{1400} \times \frac{\pi}{4}$$

$$= (1137.5 + 924.22)\,\frac{\pi}{4} = 1618.45 \text{ Nm}$$

$$\Delta E = I\ k_s\ \omega^2$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 1450}{60} = 151.76 \text{ r/s}$$

Coefficient of speed fluctuation $k_s = \frac{2+2}{100} = 0.04$

$\therefore$ $1618.45 = I \times 0.04 \times (151.76)^2$

or, $I = \frac{1618.45}{0.04 \times 23032.1} = 1.76 \text{ kg m}^2$

Example 4.6

A punching machine punches 6 holes per minute. The diameter of each hole is 4 cm and thickness of the plate is 3 cm. The stroke of the punch is 10 cm. The work done per square cm of sheared area is 600 J. The maximum speed of the flywheel at its radius of gyration is 28 m/s. Determine mass of the flywheel required so that its speed at its radius gyration does not fall below 26 m/s. Determine power of the motor required.

Solution

The sheared area of the hole = Circumference × depth

$$= \pi \times 4 \times 3 = 12\pi = 37.68 \text{ cm}^2$$

Energy required per hole E = 37.68 × 600 = 22608 J

Number of holes per minute = 6

$\therefore$ Energy required per minute = 6 × 22608

$\therefore$ Energy required per second $= \frac{6 \times 22608}{60} = 2260.8$

$\therefore$ Power of motor required $= \frac{2260.8}{1000} = 2.2608 \text{ kW} = 2.26 \text{ kW}$

Fluctuation of energy $\Delta E = I\ k_s\ \omega^2 = \frac{1}{2} I\ (\omega_1^2 - \omega_2^2)$

$$= \frac{M}{2}\ k^2\ (\omega_1^2 - \omega_2^2)$$

$$= \frac{M}{2}\ \{(k\,\omega_1)^2 - (k\,\omega_2)^2\}$$

$$= \frac{M}{2}\ (V_{\max}^2 - V_{\min}^2)$$

The fluctuation of energy

$$\Delta E = E\left(1 - \frac{\text{Thinckess of plate}}{2 \times \text{stroke}}\right)$$

$$= 22608\left(1 - \frac{3}{2 \times 10}\right) = 22608\left(1 - \frac{3}{20}\right)$$

$$= 22608\ (1 - 0.15)$$

$$= 22608 \times 0.85 = 19216.8 \text{ J}$$

$\therefore$ $19216.8 = \frac{M}{2}\ (28^2 - 26^2) = \frac{108\ M}{2} = 54\ M$

or, M = 355.87 kg.

4.6 SUMMARY

It has been observed that the usage of reciprocating engine mechanism results in fluctuation of energy. In such cases use of flywheel is necessary in order to reduce fluctuation in speed during the cycle. A prime mover of lower power can serve the purpose if a flywheel is used. For designing a flywheel, the connecting rod is replaced by a dynamically equivalent link so that the reciprocating mass can be determined more accurately.

Flywheel absorbs excess energy in the form of kinetic energy during period of harvestation and supplies it whenever the energy supplied is less during the period of starvation. In case of single cylinder four stroke IC engine, flywheel supplies energy during suction, compression and exhaust strokes and stores excess energy supplied during expansion stroke. Therefore, flywheel facilitates running of the engine and reduces fluctuations in the speed.

In case of punching machine, energy is supplied by the motor for which torque is constant but energy required is very high when punching operation is done. The flywheel supplies energy during punching and stores energy during idle time when no punching is done. This results in requirements of lower power motor and having lower fluctuation in the speed.

4.7 KEY WORDS

Dynamically Equivalent Link : It is link which has point masses at two points such that (a) total mass is same, (b) centre of gravity is at the same position, and (c) mass moment of inertia about an axis through CG remains same.

Turning Moment : It is the moment of the force at crank pin with respect to the crank centre.

Turning Moment Diagram : It is the diagram plotted with turning moment on the *Y*-axis and angle of rotation of crank for one cycle on the *X*-axis.

Fluctuation of the Energy : It is the difference between the maximum energy at a point on the mean torque line and minimum energy at another point on the mean torque line.

Coefficient of the Fluctuation Energy : It is the ratio of fluctuation of energy to the energy of the cycle.

Fluctuation of Speed : It is the difference in maximum angular speed and minimum angular speed.

Coefficient of Fluctuation of Speed : It is the ratio of fluctuation of speed to the average angular speed.

4.8 ANSWERS TO SAQs

Please refer the preceding text for all the answers to SAQs.