Answer Key
Part A

1) D'Alembert's Principle

It states that the inertia forces and couples and the external forces and torques on a body together give statical Equilibrium

$F = -m * f_g$

$C = I_g * \alpha$

2) Expressions

Velocity of the slider (v_p) $\quad v_p = r\omega\left[\sin\theta + \left(\frac{\sin 2\theta}{2n}\right)\right]$

Acceleration of the slider (a_p) $\quad a_p = r\omega^2\left[\cos\theta + \left(\frac{\cos 2\theta}{n}\right)\right]$

Angular velocity of the connecting rod (ω_{pc}) $\quad \omega_{pc} = \frac{\omega\cos\theta}{\sqrt{n^2 - \sin^2\theta}}.$

Angular acceleration of the connecting rod (α_{pc}) $\alpha_{pc} = \frac{\omega^2 \sin\theta}{n}$

3) Difference between static and dynamic force analysis

In static equilibrium the body is either at rest or moving with a constant velocity, its mean that body should have a zero linear acceleration, while in dynamic equilibrium body is rotating at a constant angular velocity or its angular acceleration would be zero.

4) ***Piston Effort***

***Force acting on the Piston*:**

$\mathbf{F = F_p - F_b - F_f}$

Crank Effort

$\boldsymbol{Ft = F/\cos\beta * (\sin(\theta+\beta))}$

Part B

<u>Problem 1</u>

In a slider crank mechanicsm the length of the crank and connecting rod are 150mm and 600mm respectively. The crank makes ana angle of 60^0 with the IDC and revolves at a uniform speed of 300 r.p.m. Find

Velocity and Acceleration of the slider

Angular velocity and Angular acceleration of the connecting rod

Given Data:

Length of the crank (r)	= 150 mm
Length of the connecting rod (l)	= 600 mm
Angle (θ)	= 60°
Speed (N)	= 300 rpm

To Find:

Velocity of the slider (v_p)

Acceleration of the slider (a_p)

Angular velocity of the connecting rod (ω_{pc})

Angular acceleration of the connecting rod (α_{pc})

Solution:

$$\omega = \frac{2\pi N}{60}$$

ω = 31.4 rad/s

n= l/r =600/150

n = 4

Velocity of the slider (v_p)

$$v_p = r\omega\left[\sin\theta + \left(\frac{\sin 2\theta}{2n}\right)\right]$$

= (0.15*31.4) (sin 60° + (sin 120°/2*4))

= 4.71 * 0.974

= 4.58 m/sec

Acceleration of the slider (a_p)

$$a_p = r\omega^2\left[\cos\theta + \left(\frac{\cos 2\theta}{n}\right)\right]$$

= ($0.15*31.4^2$) (cos 60° + (cos 120°/4))

= 147.89 * 0.375)

= 55.46 m/sec²

Angular velocity of the connecting rod (ω_{pc})

$$\omega_{pc} = \frac{\omega\cos\theta}{\sqrt{n^2 - \sin^2\theta.}}$$

= 15.7/3.91

= 4.02 rad/s

Angular acceleration of the connecting rod (α_{pc})

$$\alpha_{pc} = \frac{\omega^2\sin\theta}{n}$$

= 853.87/4

= 213.47 rad/s²

<u>Problem 2</u>

A Petrol engine has a stroke of 120mm and connecting rod is 3times of crank length.the crank rotates at 1500 r.p.m clockwise,determine:

a. Velocity and Acceleration of the slider
b. Angular velocity and Angular acceleration of the connecting rod

When the piston has travelled 1/4th of its stroke from IDC

Given Data:

Stroke Length (L)	= 120 mm
Length of the crank (r)	= L/2 = 60 mm
Length of the connecting rod (l)	= 3r = 180 mm
Speed (N)	= 1500 rpm

To Find:

Velocity of the slider (v_p)
Acceleration of the slider (a_p)
Angular velocity of the connecting rod (ω_{pc})
Angular acceleration of the connecting rod (α_{pc})

Solution:

$$\omega = \frac{2\pi N}{60}$$

ω = 157.08 rad/s

n = 3

$1/4^{th}$ of its stroke = ¼ * 180°

θ =45°

Velocity of the slider (v_p)

$$v_p = r\omega\left[\sin\theta + \left(\frac{\sin 2\theta}{2n}\right)\right]$$

= (157.08*.06) (sin 45° + (sin 90°/2*3))

= 8.235 m/sec

Acceleration of the slider (a_p)

$$a_p = r\omega^2\left[\cos\theta + \left(\frac{\cos 2\theta}{n}\right)\right]$$

= (.06*157.08²) (cos 45° + (cos 90°/3))

= 1046.83 m/sec²

Angular velocity of the connecting rod (ω_{pc})

$$\omega_{pc} = \frac{\omega\cos\theta}{\sqrt{n^2 - \sin^2\theta.}}$$

= 38.1 rad/s

Angular acceleration of the connecting rod (α_{pc})

$$\alpha_{pc} = \frac{\omega^2\sin\theta}{n}$$

= 5815.75 rad/s²

<u>Problem 3</u>

A horizontal steam engine running at 240 r.p.m has a bore of 200mm and stroke of 360mm. The piston rod is 20mm in diameter and connecting rod length is 900mm. the mass of the reciprocating parts is 7Kg and the frictional resistance is equivalent to a force of 500N. Determine the following when the crank is at 120^0 from the IDC, the mean pressure being 5000N/m² on the cover side and 100N/m² on the crank side.

Thrust on the connecting rod, Thrust on the cylinder walls

Loads on the bearings Turning moment on the crankshaft

Given Data:

Speed (N)	= 240 rpm	
Bore diameter (d)	= 200 mm	=0.2 m
Stroke Length (L)	= 360 mm	=0.36 m
Piston rod diameter (d)	= 20 mm	=0.02 m
Length of the connecting rod (l)	= 900 mm	= 0.9 m
Mass of the reciprocating Parts	= 7kg	
Frictional Resistance	= 500 N	
Angle (θ)	= 120 °	
Pressure on cover side (P1)	= 5000 N/m²	
Pressure on crank side (P2)	= 100 N/m²	

To Find:

Thrust on the connecting rod (F_c),

Thrust on the cylinder walls (F_n)

Loads on the bearings (F_r)

Turning moment on the crankshaft (T)

Solution:

$$\omega = \frac{2\pi N}{60}$$

= 25.13 rad/s

r = L/2

= .18m

n=l/r

= .9/.18

= 5

$\sin\beta = \frac{\sin\theta}{n}$ =0.173

β = 9.96°

***Force acting on the Piston*:**

$F = F_p - F_b - F_f$

Force due to Gas Pressure (F_p) = $P_1A_1 - P_2A_2$

= (5000*(π/4)*(0.2²))-(100*(π/4)*(0.2²-0.02²))

= 157.08 – 3.11

= 153.97 N

Inertia Force (F_b) = $m r \omega^2 \left[\cos\theta + \left(\frac{\cos 2\theta}{n}\right)\right]$

= 7*0.18*25.13²(cos 120° + (cos 120°/5))

= 795.71 * (-0.6)

= -477.43 N

Frictional Resistance = 500 N

F = $F_p - F_b - F_f$

= 153.97+477.43-500

= 131.4 N

Thrust on the connecting rod (F_c),

F_c = F/cos β

= 131.4/cos 9.96

= 131.4/0.98

= 134.08 N

Thrust on the cylinder walls (F_n)

Fc = F tan β

= 131.4 * tan 9.96°

=131.4/0 .18

= 730 N

Loads on the bearings (F_r)

*Fr = (F/cos*β) (cos (θ+β))

= 134.4 * (-0.64)

= -84.1 N

Turning moment on the crankshaft (T)

*Fr = (F/cos*β) (sin (θ+β)) r

= (134.4) (0.77) (0.18)

= 18.63 Nm

<u>Problem 4</u>

The crank and connecting rod of a vertical petrol engine, running at 1800 r.p.m are 60mm and 270mm respectively. The diameter of the piston is 100mm and the mass of the reciprocating parts is 1.2 kg. During the expansion stroke when the crank has turned 20^0 from the TDC, the gas pressure is 650kN/m². Determine the

- Net force on the piston
- Net load on the gudgeon pin
- Thrust on the cylinder walls

- Speed at which the gudgeon pin load is reversed in direction

Given Data:

Speed (N)	=1800 rpm	
Crank radius (r)	= 60 mm	=0.06 m
Connecting rod Length (l)	= 270 mm	=0.27 m
Diameter of Piston (d)	= 100 mm	= 0.1 m
Mass of reciprocating parts (m)	= 1.2 kg	
Angle (θ)	= 20°	
Gas Pressure	= 650kN/m²	

To Find:

- Net force on the piston (F)
- Net load on the gudgeon pin (F_c)
- Thrust on the cylinder walls (F_n)
- Speed at which the gudgeon pin load is reversed in direction (N)

Solution:

$$n = l/r$$
$$n=0.27/0.06 =4.5$$
$$\omega = \frac{2\pi N}{60}$$
$$=188.5 \text{ rad/s}$$
$$\sin\beta = \frac{\sin\theta}{n}$$
$$\beta = 4.36°$$

Force due to gas pressure: F_p

$$F_p = \text{Area} * \text{Pressure}$$
$$= (\pi/4)(d^2) * P$$
$$= (\pi/4)(0.1^2) * 650*10^3$$
$$= 5105 \text{ N}$$

Inertia Force (Fb):

$$mr\omega^2\left[\cos\theta + \left(\frac{\cos 2\theta}{n}\right)\right]$$
$$= 1.2*0.06*(188.5)^2 (\cos 20° + (\cos 40°/4.5))$$
$$\mathbf{= 2840\ N}$$

Net force on the piston (F)

$$F = F_p - F_b + mg$$
$$F = 5105 - 2840 + 1.2*9.81$$
$$\mathbf{= 2276.8\ N}$$

Net load on the gudgeon pin (F_c)

$$F_c = F/\cos\beta$$
$$= 2276.8/\cos 4.6°$$
$$\mathbf{= 2283.4\ N}$$

Thrust on the cylinder walls (F_n)

$$F_c = F\tan\beta$$
$$= 2276.8 * \tan 4.6°$$
$$\mathbf{= 173.5 N}$$

Speed at which the gudgeon pin load is reversed in direction (N)

$$F = F_p - F_b + mg = 0$$
$$= 5105 - 1.2*0.06*\omega^2\left[\cos 20° + \left(\frac{\cos 40°}{4.5}\right)\right] + 1.2*9.81$$

$0.07991\omega^2 = 5116.8$

$\omega = 253.04$

N = 2416.3 rpm