

## Answer Key Part A

### 1) Conditions for static and dynamic equilibrium

For a free body in static equilibrium, the vector sum of all forces acting on the body must be zero and the vector sum of all moments about any arbitrary point must also be zero. These conditions can be expressed mathematically as follows:

$$\sum F = 0$$

$$\sum T = 0$$

### 2) D'Alembert's Principle

It states that the inertia forces and couples and the external forces and torques on a body together give statical Equilibrium

$$F = -m \cdot f_g$$

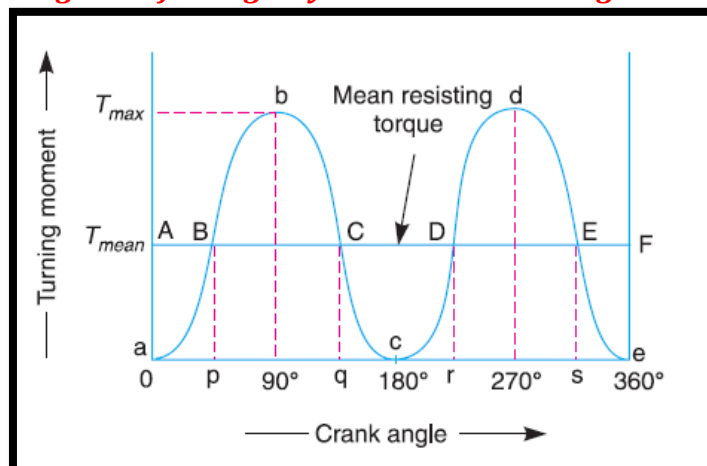
$$C = I_g \cdot \alpha$$

### 3) Coefficient of fluctuation of energy.

It is defined as the ratio of the maximum fluctuation of energy to the work done per cycle.

$$CE = \frac{\text{Maximum fluctuation of energy}}{\text{Work done per cycle}}$$

### 4) Turning moment diagram of a single cylinder double acting steam engine.



## Part B

### Problem 1

A horizontal steam engine running at 240 r.p.m has a bore of 200mm and stroke of 360mm. The piston rod is 20mm in diameter and connecting rod length is 900mm. the mass of the reciprocating parts is 7Kg and the frictional resistance is equivalent to a force of 500N. Determine the following when the crank is at  $120^\circ$  from the IDC, the mean pressure being  $5000\text{N/m}^2$  on the cover side and  $100\text{N/m}^2$  on the crank side.

Thrust on the connecting rod, Thrust on the cylinder walls  
Loads on the bearings, turning moment on the crankshaft

**Given Data:**

Speed (N)	= 240 rpm	
Bore diameter (d)	= 200 mm	=0.2 m
Stroke Length (L)	= 360 mm	=0.36 m
Piston rod diameter (d)	= 20 mm	=0.02 m
Length of the connecting rod (l)	= 900 mm	= 0.9 m
Mass of the reciprocating Parts	= 7kg	
Frictional Resistance	= 500 N	
Angle ( $\theta$ )	= 120 °	
Pressure on cover side (P1)	= 5000 N/m <sup>2</sup>	
Pressure on crank side (P2)	= 100 N/m <sup>2</sup>	

**To Find:**

Thrust on the connecting rod ( $F_c$ ),  
 Thrust on the cylinder walls ( $F_n$ )  
 Loads on the bearings ( $F_r$ )  
 Turning moment on the crankshaft (T)

**Solution:**

$$\omega = \frac{2\pi N}{60} = 25.13 \text{ rad/s}$$

$$r = L/2 = .18\text{m}$$

$$n = l/r = .9/.18 = 5$$

$$\sin \beta = \frac{\sin \theta}{n} = 0.173$$

$$\beta = 9.96^\circ$$

**Force acting on the Piston:**

$$F = F_p - F_b - F_f$$

Force due to Gas Pressure ( $F_p$ )

$$= P_1 A_1 - P_2 A_2$$

$$= (5000 * (\pi/4) * (0.2^2)) - (100 * (\pi/4) * (0.2^2 - 0.02^2))$$

$$= 157.08 - 3.11$$

$$= 153.97 \text{ N}$$

Inertia Force ( $F_b$ )

$$= m r \omega^2 \left[ \cos \theta + \left( \frac{\cos 2\theta}{n} \right) \right]$$

$$= 7 * 0.18 * 25.13^2 (\cos 120^\circ + (\cos 120^\circ / 5))$$

$$= 795.71 * (-0.6)$$

$$= -477.43 \text{ N}$$

Frictional Resistance

$$= 500 \text{ N}$$

F

$$= F_p - F_b - F_f$$

$$= 153.97 + 477.43 - 500$$

$$= 131.4 \text{ N}$$

**Thrust on the connecting rod ( $F_c$ ),**

$$F_c = F / \cos \beta$$

$$= 131.4 / \cos 9.96$$

$$= 131.4 / 0.98$$

$$= \mathbf{134.08 \text{ N}}$$

**Thrust on the cylinder walls ( $F_n$ )**

$$F_c = F \tan \beta$$

$$= 131.4 * \tan 9.96^\circ$$

$$= 131.4 / 0.18$$

$$= 730 \text{ N}$$

**Loads on the bearings ( $F_r$ )**

$$\begin{aligned} F_r &= (F/\cos\beta) (\cos(\theta+\beta)) \\ &= 134.4 * (-0.64) \\ &= -84.1 \text{ N} \end{aligned}$$

**Turning moment on the crankshaft ( $T$ )**

$$\begin{aligned} F_r &= (F/\cos\beta) (\sin(\theta+\beta)) r \\ &= (134.4) (0.77) (0.18) \\ &= 18.63 \text{ Nm} \end{aligned}$$

**Problem 2:**

The intercepted areas between the output torque curve and the mean resistance line of a turning moment diagram for a multi cylinder engine, taken in order from one end are as follows:-0.35,4.10,-2.85,3.25,-3.35,2.60,-3.65,2.85,-2.6 sq cm. The diagram drawn into a scale of 1cm=700Nm and 1cm = 45°. The engine speed is 900rpm and the fluctuation of speed is not to exceed 2% of the mean speed. Find the suitable diameter and cross section of the flywheel rim if the safe centrifugal stress is limited to 7MPa. The density of the material is 7200kg/m<sup>3</sup>. The rim is rectangular with the width 2 times the thickness. Neglect the effect of arms.

**Solution:**

Let Flywheel KE at a = E

at a	= E	
at b	= E-0.35	<b>E-0.35 (Min Energy)</b>
at c	= E-0.35+4.10	E+3.75
at d	= E+3.75-2.85	E+0.9
at e	= E+0.9+3.25	<b>E+4.15 (Max Energy)</b>
at f	= E+4.15-3.35	E+0.8
at g	= E+0.8+2.60	E+3.4
at h	= E+3.4-3.65	E-0.25
at i	= E+2.85-0.25	E+2.6
at j	= E+2.6-2.6	E

Max Energy : E+4.15

Min Energy : E-0.35

Maximum Fluctuation of energy:

$$\begin{aligned} \Delta E &= \text{Max Energy} - \text{Min Energy} \\ &= E+4.15 - (E-0.35) \end{aligned}$$

$$\Delta E = 4.5 \text{ cm}^2$$

**Scale:**

$$1 \text{ cm} = 700 \text{ Nm}$$

$$1 \text{ cm} = 45^\circ$$

$$1 \text{ mm}^2 \text{ in turning moment diagram}$$

$$= (45 * \pi / 180) * 700$$

$$= 549.78 \text{ Nm}$$

$$4.5 \text{ cm}^2 = 2474.01 \text{ Nm}$$

$$\sigma = \rho v^2$$

$$7 * 10^6 = 7200 * v^2$$

$$v = 31.18 \text{ m/s}$$

$$v = \pi D N / 60$$

$$31.18 = (\pi * D * 900) / 60$$

$$\begin{aligned}
D &= 0.66\text{m} \\
\omega &= 2\pi N/60 \\
&= (2*\pi*900)/60 \\
&= 94.25\text{ rad/s} \\
\Delta E &= I\omega^2 C_s \\
&= mk^2\omega^2 C_s \\
2474 &= (m*0.33^2*94.25^2*18) \\
m &= 0.14\text{kg} \\
m &= \pi D A \rho \\
&= \pi * D * b * t * \rho \\
0.14 &= \pi * 0.66 * 2t^2 * 7200 \\
\mathbf{t} &= \mathbf{0.218\text{mm}} \\
\mathbf{b} &= \mathbf{0.436\text{mm}}
\end{aligned}$$

### Problem 3:

A riveting machine is driven by a constant torque 3 kW motor. The moving parts including the flywheel are equivalent to 150 kg at 0.6 m radius. One riveting operation takes 1 second and absorbs 10 000 N-m of energy. The speed of the flywheel is 300 r.p.m. before riveting. Find the speed immediately after riveting. How many rivets can be closed per minute?

#### Given:

$$\begin{aligned}
P &= 3\text{ kW;} \\
m &= 150\text{ kg;} \\
k &= 0.6\text{ m;} \\
N_1 &= 300\text{ r.p.m. or } \omega_1 = 2\pi \times 300/60 = 31.42\text{ rad/s}
\end{aligned}$$

#### Speed of the flywheel immediately after riveting

Let  $\omega_2$  = Angular speed of the flywheel immediately after riveting.

We know that energy supplied by the motor,

$$\begin{aligned}
E_2 &= 3\text{ kW} \\
&= 3000\text{ W} \\
&= 3000\text{ N-m/s ((ie) } 1\text{ W} = 1\text{ N-m/s)}
\end{aligned}$$

But energy absorbed during one riveting operation which takes 1 second,

$$E_1 = 10\,000\text{ N-m}$$

Therefore Energy to be supplied by the flywheel for each riveting operation per second or the maximum fluctuation of energy,

$$\begin{aligned}
\Delta E &= E_1 - E_2 \\
&= 10\,000 - 3000 \\
&= 7000\text{ N-m}
\end{aligned}$$

We know that maximum fluctuation of energy ( $\Delta E$ ),

$$\begin{aligned}
\Delta E &= \left(\frac{1}{2}\right) * I^2 * \omega^2 \\
&= (1/2)(mk^2)(\omega_1^2 - \omega_2^2) \\
&= (1/2)(150*0.6^2)(31.42^2 - \omega_2^2) \\
\omega^2 &= 987.2 - 7000/27 \\
&= 728\text{ or}
\end{aligned}$$

$$\omega_2 = 26.98\text{ rad/s}$$

Corresponding speed in r.p.m.,

$$\begin{aligned}
N_2 &= 26.98 \times 60 / 2\pi \\
&= \mathbf{257.6\text{ r.p.m. Ans.}}
\end{aligned}$$

#### Number of rivets that can be closed per minute

Since the energy absorbed by each riveting operation which takes 1 second is 10 000 N-m, therefore, number of rivets that can be closed per minute,

$$= (E2/E1)*60$$

$$= (3000/10000)*60$$

$$= \mathbf{18 \text{ rivet Ans.}}$$

#### **Problem 4:**

Four masses A, B, C and D as shown below are to be completely balanced.

	A	B	C	D
Mass (kg)	-	30	50	40
Radius (mm)	180	240	120	150

The planes containing masses B and C are 300mm apart. The angles between planes containing B and C is  $90^\circ$ , B and C makes angles of  $210^\circ$  and  $120^\circ$  respectively with D in the same sense. Find,

- The magnitude and the angular position of mass A
- The position of planes A and D

#### **Given Data:**

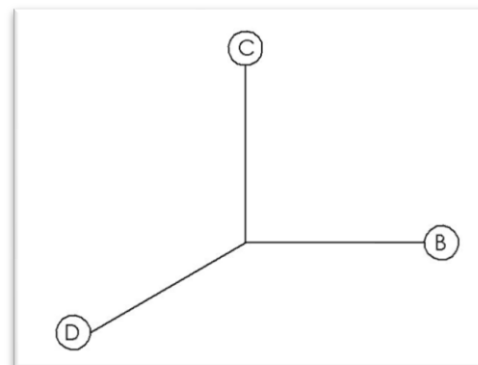
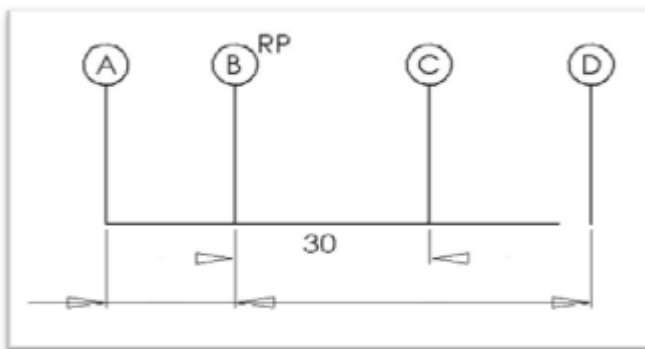
$m_a = - \text{kg}$	$r_1 = 180 \text{ mm}$	$\theta_1 = ?^\circ$
$m_b = 30 \text{ kg}$	$r_2 = 240 \text{ mm}$	$\theta_2 = 0^\circ$
$m_c = 50 \text{ kg}$	$r_3 = 120 \text{ mm}$	$\theta_3 = 90^\circ$
$m_d = 40 \text{ kg}$	$r_4 = 150 \text{ mm}$	$\theta_4 = 210^\circ$

#### **To Find:**

Magnitude of A =  $m_a$   
 Angular Position of A =  $\theta_A$   
 Position of A  
 Position of D

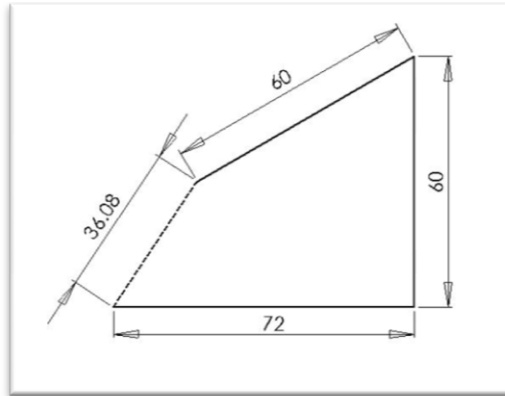
#### **Solution:**

#### **Plane Diagram & Space Diagram:**



Plane	Mass (kg)	Radius (m)	Force/ $\omega^2$ (mr) kg.m	Distance From RP (m)	Couple/ $\omega^2$ (kg.m <sup>2</sup> )
A	$m_a$	0.18	$0.18 m_a$	-y	$-0.18 m_a y$
B(RP)	30	0.24	7.2	0	0
C	50	0.12	6	0.3	1.8
D	40	0.15	6	x	6x

#### **Force Polygon:**



$$0.18m_a = 36.08/10$$

$$m_y = 3.608/0.18$$

$$\underline{m_y = 20.04 \text{ kg}}$$

$$\underline{\theta_y = 56.26 + 180^\circ}$$

$$\underline{\theta_y = 236.26^\circ}$$

**Couple Polygon:**