CHAPTER-3
GOVERNORS

3.1 Function of Governors

The function of the governor must be carefully distinguished from that of the flywheel. The former is required to maintain, as closely as possible, a constant mean speed of rotation of the crankshaft over long periods during which the load on the engine may vary. The latter serves to limit the inevitable fluctuations of speed during each cycle, which arise from the fluctuations of turning moment on the crankshaft. On the other hand the governor exercises no control over the cyclical fluctuations of the speed, while on the other hand the flywheel has no effect on the mean speed of rotation.

If the load on the engine is constant, the mean speed of rotation will be constant from cycle to cycle. However, if the load changes, the mean speed will also change while the output of the engine is not adjusted to the new demand. It is the purpose of the governor to make this adjustment automatically.

It is, of course, desirable that the energy supplied to the engine should be altered by exactly the right amount immediately the change of load takes place. But if the adjustment of the supply of the energy to the engine is to be carried out automatically, use has to be made of the tendency for the mean speed of rotation to change as the load changes. In other words, a change of speed must take place before the energy supplied to the engine can be automatically adjusted to the new load. Hence it follows that the mean speed of rotation will tend to increase continuously as the load on the engine decreases. The governor and the mechanism, which it operates should be so designed that this increases of the mean speed of rotation is as small as possible. In any actual engine the problem is complicated by the fact that the load may change immediately after a fresh charge has been supplied to the engine cylinder and, since the governor cal only affect the charge admitted during the next and succeeding cycles, some lag between the change of load and the change of engine output is inevitable.

3.2 Types of Governors

Governors are generally of one of two types, either (a) centrifugal or (b) inertia.

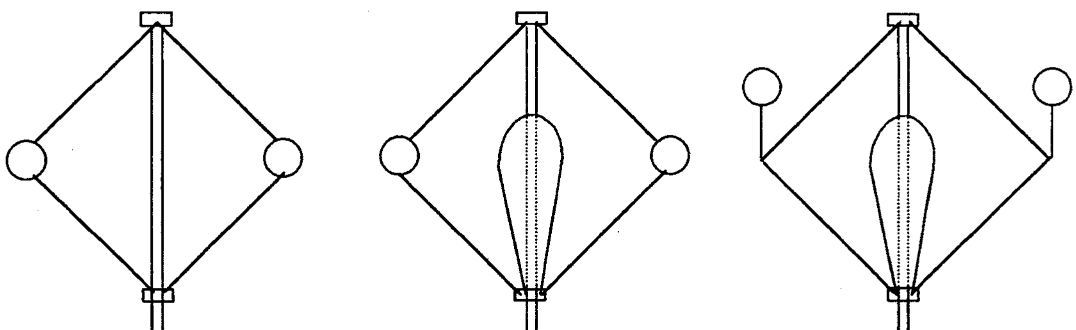
In the first type two or more masses termed the governor balls are caused to revolve about the axis of the shaft, which is driven through suitable gearing from the engine crankshaft. Each ball is acted upon by a force, which acts in the radially inward direction, and is provided by a dead weight, a spring or a combination of the two. This force is termed the controlling force and it must increase in magnitude as the distance of the ball from the axis rotation increases. When the governor balls are revolving at a uniform speed, the radius of rotation will clearly be such that the outward inertia or centrifugal force is just balanced by the inward controlling force. If the speed of rotation now increases owing to a decrease of load on the engine, the governor balls will move outward until the centrifugal force is again balanced by the controlling force. Conversely, if the speed of rotation decreases owing to the increase of load on the engine, the governor balls will move inward until the centrifugal force is again balanced by the controlling force. This movement of the balls is transmitted by the governor mechanism to the valve, which control the amount of the energy supplied to the engine, so that movement in the outward direction reduces the valve opening and movement in the inward direction increases the valve opening.

Governors of the second type operate on a different principle. The governor balls are so arranged that the inertia force caused by an angular acceleration or retardation of the governor shaft tend to alter their positions. The amount of the displacement of the governor balls caused by inertia forces is controlled by suitable springs and, through the governor mechanism, alters the amount of energy supplied to the engine. The obvious advantage of this type of governor lies in its more rapid response to the effect of the change of load, since the displacement of the balls is determined by rate of change of speed of rotation, as distinct from the actual change of speed of rotation, such as is required in governors of the first type. This advantage is offset, however, by the practical difficulty of arranging for the complete balance of the revolving parts of the governor. For this reason centrifugal governors are much more frequently used than are inertia governors, and only the former type will be dealt with here.

3.3 Centrifugal Governors

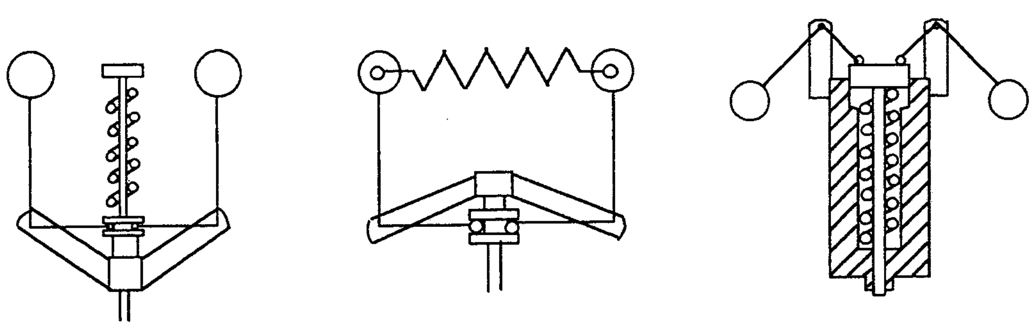
Centrifugal governors may be divided into (a) gravity loaded-controlled governors as shown in fig-3.1 and (b) spring loaded-controlled governors as shown in fig-3.2.

In the gravity loaded-controlled governors, an equation of equilibrium is obtained by taking moments of forces about instantaneous center, I, of the lower link, thus eliminating the tension in the upper link and the side thrust at the sleeve. In the spring loaded-controlled governors moment are taken about the fulcrum of the bell crank levers, eliminating the reaction at that point, and in the type shown in fig-3.2 (c) moments are taken about the instantaneous center, I, for the upper bell-crank arm to eliminate the reaction between the roller and the top of the spindle.



(a) (b) (c)

Fig-3.1



(a) (b) (c)

Fig-3.2

3.4 Gravity Loaded Controlled Governors

(a) Watt Governor

This type of governor is shown in fig-3.1 (a). It is the original form of governor as used by Watt on some of his early steam engines. In this type of governor, each ball is attached to an arm, which is pivoted on the axis of rotation. The sleeve is attached to the governor balls by arms, pin-jointed at both ends, and is free to slide along the governor shaft.

The upper arm may be suspended from the vertical spindle in three ways as shown in fig-3.3.

(i) From the axis of the spindle as shown in fig-3.3 (a).

(ii) From a point attached to a collar on the spindle so that the arm produced intersects the spindle as shown in fig-3.3 (b).

(iii) From a point to a collar so that the arm crosses the spindle as shown in fig 3.3(c).

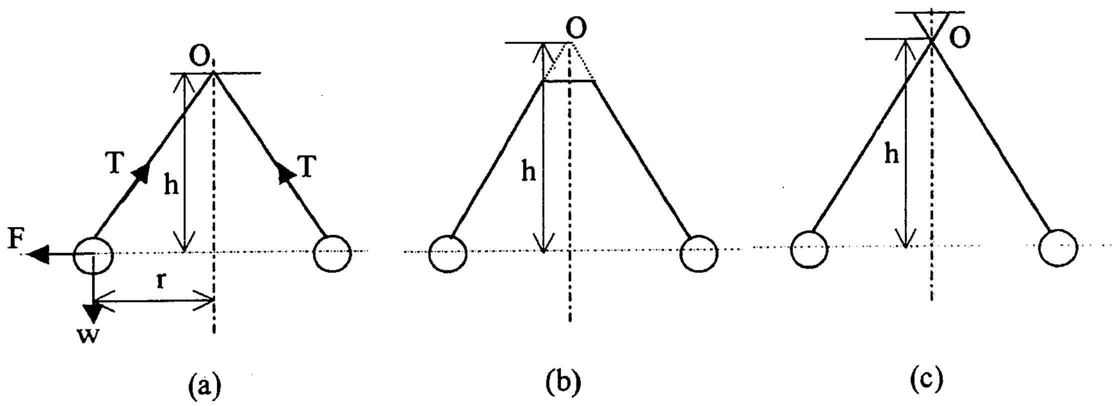


Fig-3.3

The height of the governor, which is donated by 'h' in figure, is the distance from the center of the mass to the point of intersection between the arm and the axis of the spindle.

Let 'w' be the weight of the ball, 'T' the tension in the arm and 'F' the centrifugal force when the radius to the center of the ball is 'r' and the angular velocity of the arm and the ball about the spindle axis is 'ω'.

For the simplified analysis, which follows, the weights of the sleeve, the upper ball arms, the lower links and friction are all neglected. As the weight of the lower arms and sleeve is neglected, the tensions in the lower links are negligible and hence only three forces are acting on each rotating ball.

(i) The weight 'w' acting vertically downwards

(ii) The centrifugal force '$F = \frac{w}{g}\omega^2 r$' acting radially outwards

(iii) The tension 'T' in the upper arm.

Taking moment about O, the point of intersection of the arm and the axis of the spindle, for the forces acting on the governor balls, we get

$$\frac{w}{g}\omega^2 r \times h = w \times r$$

$$h = \frac{g}{\omega^2} \qquad \text{(i)}$$

The equation (i) shows that neither the weight of the balls nor the length of the supporting arms has any influence on the height of the governor. It varies inversely as the square of the speed.

When 'g' is in cm/s^2 and 'ω' is in radian/s, then 'h' is in cm.

Let 'N' be the speed in rpm, then

$$\omega = \frac{2\pi N}{60} = \frac{\pi N}{30}$$

$$\therefore h = \frac{900g}{\pi^2 N^2} = \frac{900 \times 981}{\pi^2 N^2} = \frac{89560}{N^2} \text{ cm.}$$

Since the height of the governor is inversely proportional to the square of the speed it is small at high speeds and at such speeds the change in height corresponding to a small change in speed is insufficient to enable a governor of the Watt type to operate the mechanism to give the necessary change in the fuel supply or steam supply.

From the table given below it can be seen that the height diminishes very rapidly as the speed of rotation increases.

N (rpm)	40	60	80	100	120	150	220
h (cm)	55.98	24.88	13.98	8.96	6.22	3.98	2.24

Thus, this governor is suitable only for low speeds of rotation not exceeding 75 rpm. It might then be suggested that a speed reduction gear between engine shaft and the governor spindle would allow this governor to be used with higher speed engines. However, it should be noted that this is not a satisfactory remedy.

(b) Porter Governor

The type of governor, which is illustrated at fig-3.1 (b), is known as the Porter governor. The only respect in which it differs from the Watt governor is in the use of a heavily weighted sleeve. The additional downward force increases the speed of revolution required to enable the balls to rise to any pre-determined level.

Let 'w' be the weight of each ball and 'W' be the weight of the central load. T_1 be the tension in the upper arm and T_2 the tension in the suspension link. α and β be the inclinations to the vertical of the upper arm and suspension links respectively. The weight of arms and weight of suspension links and the effect of friction to the movement of the sleeve are neglected.

There are several ways of determining the relation between the height 'h' and the speed 'ω'. In this chapter, two methods are used to derive the relation.

(i) Instantaneous Center Method

Consider the equilibrium of the forces acting on the suspension link 'AC', which is shown in fig-3.4. These forces are 'F, w and T_1 at C and $\frac{W}{2}$ and Q at A. The equation connecting 'F, w and W' is derived by taking moment about I, the point of intersection of the lines of action of forces T_1 and Q. This point of intersection I is also the instantaneous center of the link AC. The point I lies at the point of intersection of BC produce and a line drawn through A perpendicular to the axis of the governor spindle.

Taking moment about I,

$$F \times CD = w \times ID + \frac{W}{2}(ID + DA)$$

$$F = w \times \frac{ID}{CD} + \frac{W}{2}\left(\frac{ID}{CD} + \frac{DA}{CD}\right)$$

$$= w \tan\alpha + \frac{W}{2}(\tan\alpha + \tan\beta)$$

$$= \left\{\frac{W}{2}\left(1 + \frac{\tan\beta}{\tan\alpha}\right) + w\right\}\tan\alpha$$

$$= \left\{\frac{W}{2}(1 + k) + w\right\}\tan\alpha$$

where $k = \frac{\tan\beta}{\tan\alpha}$.

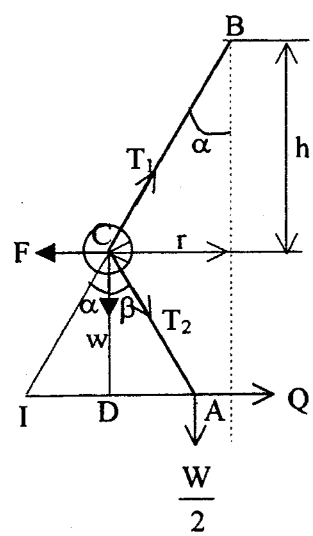


Fig-3.4

If 'h' be the height of the governor, then $\tan\alpha = \frac{r}{h}$. Further, we have $F = \frac{w}{g}\omega^2 r$.

Therefore, we get

$$\frac{w}{g}\omega^2 r = \left[\frac{W}{2}(1+k)+w\right]\frac{r}{h} \qquad \text{(or)}$$

$$\omega^2 = \left[\frac{\frac{W}{2}(1+k)+w}{w}\right]\frac{g}{h} \qquad \text{(i)}$$

When the length of the arms and the suspension links are of equal length and the axis of the joints at B and A either intersect the governor spindle or are at equal distances from the governor spindle the value 'k' is equal to 1 and the equation (i) reduces to the form

$$\omega^2 = \left(\frac{W+w}{w}\right)\frac{g}{h} \qquad \text{(ii)}$$

When the lengths of the arms are unequal and the axes of the joints at B and A are at different distances from the governor spindle the k will have a different value for each radius of rotation of the governor balls, This value of 'k' can be best found by calculating the value of α and β. It should be noted that when 'k' is not equal to 1, its value changes as the height of the governor changes.

For the simple Watt governor, the weight of the sleeve W is negligible and we have either from equation (i) or (ii) the relation $\omega^2 = \frac{g}{h}$ which has derived earlier.

(ii) Equilibrium Method

The governor sleeve, which is loaded by the weight W is in equilibrium under a system of three forces, W the load on the sleeve and the tensions T_2 in the two lowered suspension links. As the system of forces is in equilibrium, the force triangle drawn for these forces must be a closed one as shown in fig-3.5 (a).

The pin joint C between the upper arm and the lower suspension link must be in equilibrium under the action of the four forces as under:

(i) The weight of the ball 'w'

(ii) Radially outwards acting centrifugal force $F = \frac{w}{g}\omega^2 r$

(iii) Tension T_1 in the upper arm

(iv) Tension T_2 in the lower suspension link.

These four forces must form a closed polygon as shown in fig-3.5 (b).

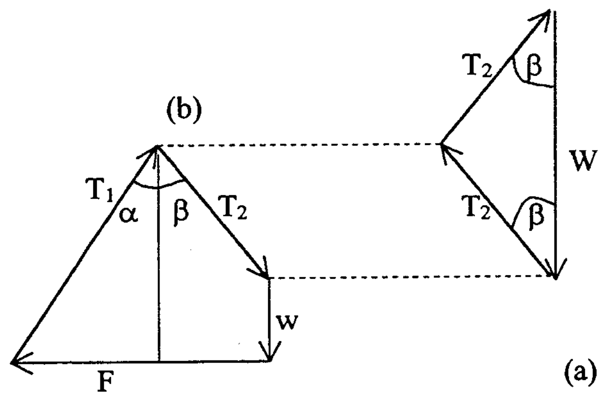


Fig-3.5

From force triangle for the sleeve, we get

$$W = 2\,T_2 \cos\beta \qquad \text{(or)} \qquad T_2 = \frac{W}{2\cos\beta} \qquad \text{(iii)}$$

From the polygon of forces on the ball, we have

$$T_1 \cos\alpha = T_2 \cos\beta + w \qquad \text{(resolving vertically)} \qquad \text{(iv)}$$

Resolving horizontally,

$$F = T_1 \sin\alpha + T_2 \sin\beta \qquad \text{(v)}$$

From equation (iv)

$$T_1 = \frac{\frac{W}{2} + w}{\cos\alpha}$$

When the value of T_1 and T_2 are substituted in the equation (v),

$$F = \left(\frac{W}{2} + w\right)\tan\alpha + \frac{W}{2}\tan\beta$$

$$= \left[\frac{W}{2}(1+k) + w\right]\tan\alpha \qquad \text{(v)}$$

where $k = \dfrac{\tan\beta}{\tan\alpha}$.

By substituting the value of $\tan\alpha$ and F, equation (i), which is derived earlier, can be done.

Friction at the sleeve

The friction resistances of the various joints of the governor itself and of the gear, which the governor has to operate, may be reduced to single force 'R' acting on the sleeve in a direction opposite to that of its motion. Since friction always opposes the motion, the friction may be regarded as an additional load at the sleeve when the speed is increasing and the sleeve is being lifted. The central load will be altered from W to W+R.

When the speed is decreasing, the sleeve will be descending and the friction force will be in upward direction. The effect of friction at the sleeve will be to reduce the central load from W to W-R.

Example (1)

The upper arms of a Porter governor are pivoted on the axis of rotation and the lower arms are attached to sleeve at distance of 3.76 cm from the axis. The length of the arms and suspension links are 30 cm. The weight of each ball is 6 kg and the load of the sleeve is 48 kg. If the extreme radii of rotation of the governor balls are 20 cm and 25 cm, find the corresponding equilibrium speeds.

Suffixes 1 and 2 are used to distinguish between the maximum and minimum speed of rotation.

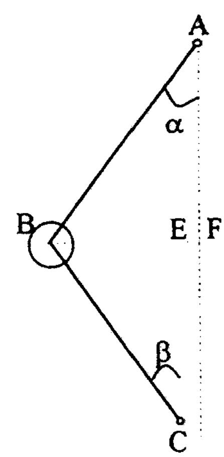


Fig-3.6

W = weight of sleeve = 48 kg.

w = the weight of governor ball = 6 kg.

Refer to fig-3.6

At maximum radius:

$$AF = \sqrt{30^2 - 25^2} = 16.6\,\text{cm}$$

$$CE = \sqrt{30^2 - 21.24^2} = 21.2\,\text{cm}$$

$$k_1 = \frac{\tan\beta_1}{\tan\alpha_1} = \frac{BE}{CE} \times \frac{AF}{BF} = \frac{21.24}{21.2} \times \frac{16.6}{25} = 0.665$$

$$\omega_1^2 = \left[\frac{\frac{W}{2}(1+k_1) + w}{w}\right]\frac{g}{h_1}$$

$$= \left[\frac{\frac{48}{2}(1 + 0.665) + 6}{6} \right] \frac{981}{16.6}$$

$$\omega_1 = 21.2 \text{ rad/s}^2$$

$$N_1 = \frac{21.2 \times 60}{2\pi} = 202 \text{ rpm (Ans)}$$

At minimum radius:

$$AF = \sqrt{30^2 - 20^2} = 22.4 \text{ cm}$$

$$CE = \sqrt{30^2 - 16.24^2} = 25.2 \text{ cm}$$

$$k_2 = \frac{\tan \beta_2}{\tan \alpha_2} = \frac{BE}{CE} \times \frac{AF}{BF} = \frac{16.24}{25.2} \times \frac{22.4}{20} = 0.72$$

$$\omega_2{}^2 = \left[\frac{\frac{W}{2}(1 + k_2) + w}{w} \right] \frac{g}{h_2}$$

$$= \left[\frac{\frac{48}{2}(1 + 0.72) + 6}{6} \right] \frac{981}{22.4}$$

$\omega_2 = 18.6 \text{ rad/s}^2$ (or) $N_2 = 178.5$ rpm (Ans)

Example (2)

Fig-3.7 (a) shows a Porter governor for which the speed range can be varied by means of an auxiliary spring S. The spring force is transmitted to the sleeve by the arm AB, which is pivoted at A. The weight of each ball is 0.36 kg and the lengths of the links and arms are 7.5 cm. The sleeve carries a weight of 1 kg. The sleeve begins to rise when the balls revolve at 200 rpm in a circle of 7.5 cm radius. The speed of the governor is not to exceed 225 rpm when the sleeve has risen 1 cm from its original position.

Determine the necessary stiffness of the auxiliary spring S and the tension in the upper arms and lower links when the sleeve begins to rise from its equilibrium position for a speed of 200 rpm.

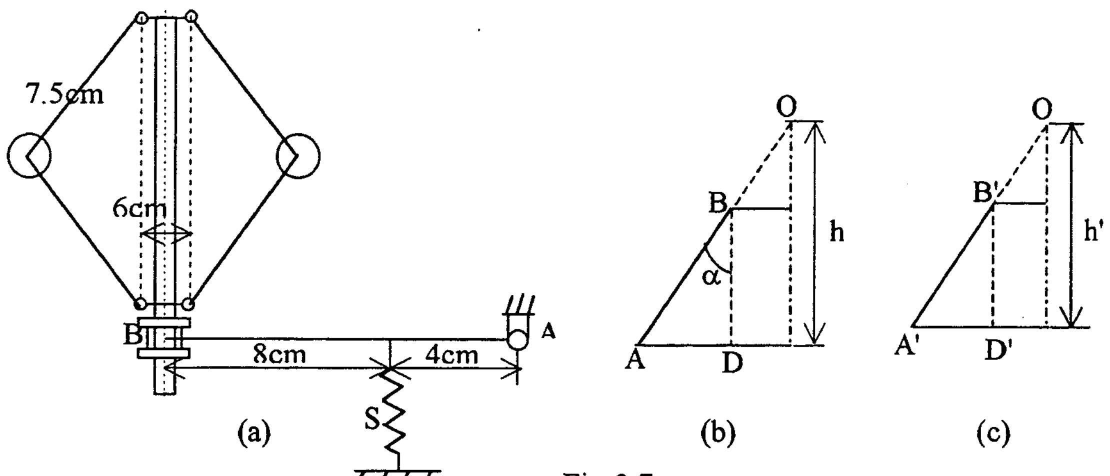


Fig-3.7

If P be the force exerted by the auxiliary spring then the force exerted on the sleeve by the spring will be $\frac{P \times 4}{8+4} = \frac{P}{3}$.

Therefore total load on the sleeve will be $(1 + \frac{P}{3})$ kg.

As the lengths of the arms and the suspension links are equal and they are attached at equal distances from the axis of rotation of the spindle, the value of k is equal to 1, and the height is given by the equation $h = \left(\frac{W+w}{w}\right)\frac{g}{\omega^2}$.

When the speed is 200 rpm, the radius of rotation of the ball path is 7.5 cm. From fig-3.7 (b), the height of the governor balls for this position can be obtained by similar triangles.

$$AD = 7.5 - 3 = 4.5$$

$$BD = \sqrt{7.5^2 - 4.5^2} = 6 \text{ cm}$$

$$h = \frac{7.5}{4.5} \times 6 = 10 \text{ cm}$$

$$\omega = \frac{200 \times 2\pi}{60} = 21 \text{ rad/s}$$

$$10 = \left[\frac{1 + \frac{P}{3} + 0.36}{0.36}\right]\frac{981}{(21)^2}$$

$\therefore$ P = 0.75 kg.

When the speed of rotation is 225 rpm, the radius of rotation of the ball path can be obtained with the help of fig-3.7 (c).

The travel of the sleeve is 1 cm. The vertical movement of the ball is $\frac{1}{2}$ cm.

∴ The distance B'D' = $6 - \frac{1}{2} = 5.5$ cm.

$$A'D' = \sqrt{7.5^2 - 5.5^2} = 5.1\,\text{cm}$$

Radius of rotation of ball path = 5.1 + 3 = 8.1 cm.

$$\text{The height of the governor} = h' = \frac{8.1}{5.1} \times 5.5 = 8.74\,\text{cm}.$$

$$\omega' = \frac{225 \times 2\pi}{60} = 23.6 \text{ rad/s}$$

If P' be the load in the spring when the speed of rotation is 225 rpm, then

$$8.74 = \frac{981}{(23.6)^2}\left[\frac{1 + \frac{P'}{3} + 0.36}{0.36}\right]$$

$$P' = 1.27\text{ kg}$$

When the sleeve rises by 1 cm, the spring extends by the amount $\frac{4}{12} \times 1 = \frac{1}{3}$ cm.

$$\text{Spring stiffness} = \frac{(1.27 - 0.75)}{\frac{1}{3}} = 1.44\,\text{kg/cm}.$$

At 200 rpm, the load on the sleeve is $1 + \frac{0.75}{3} = 1.25$ kg. From fig-3.7 (a), the sum of the vertical components of the tensions in the suspension link equals the load on the sleeve.

$$\therefore \cos\alpha = \cos\beta = \frac{BD}{AB} = \frac{6}{7.5} = 0.8$$

$$\text{The tension in the suspension link} = T_2 = \frac{W}{2\cos\alpha} = \frac{1.2}{2 \times 0.8} = .78\,\text{kg}.$$

$$\text{The tension in the suspension link} = T_1 = \frac{\frac{W}{2} + w}{\cos\alpha} = \frac{\frac{1.25}{2} + 0.36}{0.8} = 1.23\,\text{kg}.$$

(c) Proell Governor

Fig-3.1(c) shows a type of Proell governor. This governor is similar to the Porter governor except that the revolving balls are attached to the extensions of the lower links. This has the effect of reducing the change of speed necessary for a given sleeve movement. In other words the governor is made more sensitive.

The action of this governor is again similar to that of the other governors described earlier. The analysis of the Proell governor can be done by considering the equilibrium of the lower arm, which is referred fig-3.8.

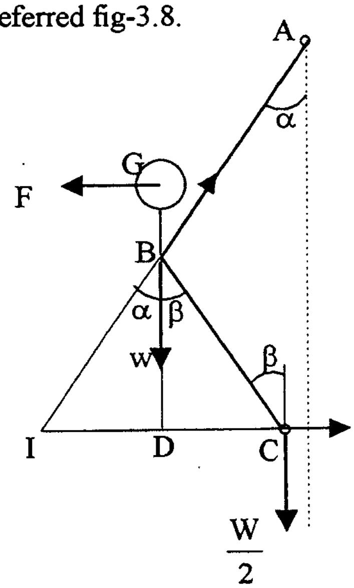


Fig-3.8

There are five forces acting on the lower link:

(i) The centrifugal force F, acting radially outwards, through the center of the gravity of the ball

(ii) The weight 'w', acting vertically downwards through the center of gravity of the ball

(iii) The pull $\frac{W}{2}$ at C acting vertically downwards

(iv) The tension T_1 along the length of the link AB

(v) Reaction at C along a line at right angles to the axis of the governor spindle.

The instantaneous center of the lower suspension link BC lies at the point of intersection of AB produced and a line drawn through C perpendicular to the axis of the governor spindle. It is assumed that the extension BG of the lower suspension link BC is vertical for the given configuration.

Take moment about I, the instantaneous center of the lower suspension link. The tension T_1 and the reaction at C give no moment. Therefore,

$$F \times DG = w \times ID + \frac{W}{2} \times (ID + DC) \qquad \text{(i)}$$

Dividing both sides by BD,

$$F \times \frac{DG}{BD} = w \times \frac{ID}{BD} + \frac{W}{2}\left[\frac{ID}{BD} + \frac{DC}{BD}\right]$$

$$= w \tan\alpha + \frac{W}{2}[\tan\alpha + \tan\beta]$$

$$= \left[w + \frac{W}{2}\right]\tan\alpha + \frac{W}{2}\tan\beta$$

$$\therefore F = \frac{BD}{DG}\left\{\left[w + \frac{W}{2}\right]\tan\alpha + \frac{W}{2}\tan\beta\right\} \qquad \text{(ii)}$$

Let $\dfrac{\tan\beta}{\tan\alpha} = k$

$$\therefore F = \frac{BD}{DG}\left\{\frac{W}{2}(1+k) + w\right\}\tan\alpha \qquad \text{(iii)}$$

But, $\tan\alpha = \dfrac{r}{h}$ and $F = \dfrac{w}{g}\omega^2 r$ (iv)

Substituting the values given by equation (iv) in equation (iii),

$$\frac{w}{g}\omega^2 r = \frac{BD}{DG}\left\{\frac{W}{2}(1+k) + w\right\}\frac{r}{h}$$

$$\omega^2 = \frac{g}{h} \times \frac{BD}{DG}\left\{\frac{\frac{W}{2}(1+k) + w}{w}\right\} \qquad \text{(v)}$$

Thus, the effect of placing the ball at G, instead of at the pin joint B is to reduce the equilibrium speed for given values of the height of the governor, the weight of the ball and the weight of the sleeve. Hence in order to give the same equilibrium speed for the given height and the weight of the sleeve, the smaller ball is required in Proell governor than that in Porter governor.

Example (3)

A governor of the Proell type is shown diagrammatically with certain dimensions in fig-3.9 (a). The central load acting on the sleeve weighs 25 kg and the two rotating balls each weigh 3.1 kg. When the governor sleeve is in mid position, the arm AB of the cranked lever is vertical and the radius of rotation of the ball path is 17.5 cm.

If the governor speed is to be 160 rpm when in mid position, find the length of the arm AB.

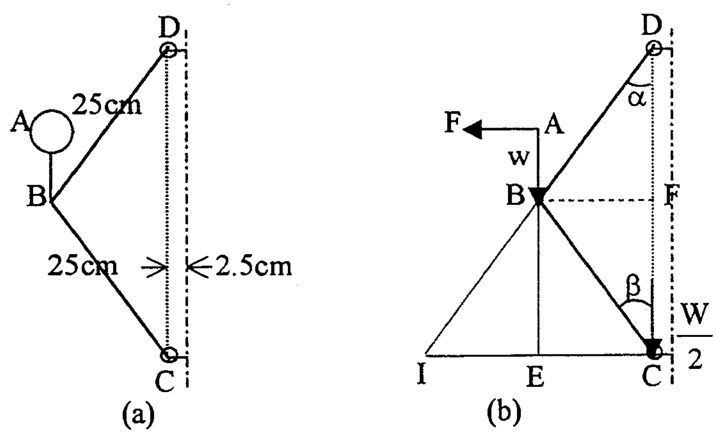


Fig-3.9

BF = 17.5 – 2.5 = 15 cm

BD = 25 cm. Similarly EC = IE = 15 cm.

$$\therefore FD = \sqrt{25^2 - 15^2} = 20cm = FC = BE$$

Taking moment about I, in fig-3.9 (b),

$$F \times AE = \frac{25}{2} \times 30 + 3.1 \times 15$$

$$F \times AE = 421.5$$

$$F = \frac{w}{g}\omega^2 r = \frac{3.1}{981} \times 17.5 \times \left(\frac{2\pi \times 160}{60}\right)^2 = 15.5\,kg$$

$$AE = \frac{421.5}{15.5} = 27.2\,cm$$

$$AB = AE - BE$$

$$= 27.2 - 20$$

$$= 7.2\ cm.(Ans)$$

3.5 Spring Loaded Controlled Governors

In spring loaded controlled governors the control of speed is affected either wholly or in part by means of springs. Some of the representative of spring loaded controlled governors are shown in fig-3.2.

The spring loaded controlled governors posses the following advantages over the gravity loaded controlled governors.

(i) The spring loaded controlled governors may be operated at very high speeds.

(ii) With proper proportioning the spring loaded controlled governors can be made both powerful and capable of very closed regulation.

(iii) It can be much smaller in over all size.

(iv) As it does not depend on gravity for its action, it may revolve about a horizontal, vertical or inclined axis.

In spring loaded controlled governors the spring may be placed upon the axis of rotation or they may be transverse as shown in fig-3.2.

(a) Spring loaded Controlled Governor of the Hartnell Type

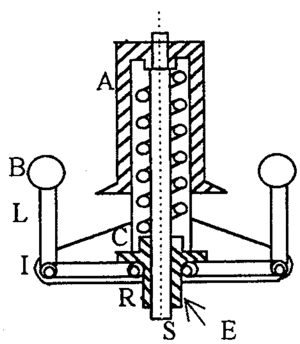


Fig-3.10

Fig-3.10 shows spring loaded controlled governor of Hartnell type. Two bell crank levers L are mounted on pins I, carried by the frame A, which is attached to the rotating spindle S. Each lever carries a ball B at the end of one arm and a roller R the end of the other. The centrifugal forces of the balls cause the rollers R to press against the collar C on the sleeve E. The upward pressure of the rollers on the collar of the sleeve is balanced by the downward thrust of the helical spring, which is in compression. The angle of the bell crank lever is usually 90° but in practice it may be grater.

Let w be the weight of each ball, S the spring force exerted on the sleeve, k the stiffness of the spring, ω the speed of rotation, r the radius of rotation, a and b the lengths of the vertical and horizontal arms of the bell crank lever and F the centrifugal force on the ball.

By taking moment about the fulcrum of the lever, neglecting the effect of pull of gravity on the governor balls and arms,

$$F \times a = \frac{S}{2} \times b$$

or $$S = 2F\frac{a}{b} \qquad \text{(i)}$$

It is assumed that the arms are mutually perpendicular and the lines of action of forces are at right angles to the arm.

Let the suffixes 1 and 2 denote the values of maximum and minimum radii respectively. Then at maximum radius

$$S_1 = 2F_1\frac{a}{b} \qquad \text{(ii)}$$

At minimum radius, $$S_2 = 2F_2\frac{a}{b} \qquad \text{(iii)}$$

$$\therefore S_1 - S_2 = 2\frac{a}{b}(F_1 - F_2)$$

Let θ be the angular movement of the bell crank lever from the position of minimum radius to the position of the maximum radius, then

$$(r_1 - r_2) = a\theta \qquad \text{(iv)}$$

If h be the lift of the sleeve, then

$$h = b\theta \qquad \text{(v)}$$

Dividing equation (v) by (iv),

$$\frac{h}{r_r - r_2} = \frac{b}{a} \qquad \text{(or)}$$

$$h = \frac{b}{a}(r_1 - r_2) \qquad \text{(vi)}$$

The difference in the forces exerted by the compressed spring in the two positions is $S_1 - S_2$; therefore, the force per unit compression is known as the stiffness of the spring. The stiffness of the spring is denoted by k.

$\therefore$ Stiffness of the spring $= k = \frac{S_1 - S_2}{h} = 2\left(\frac{a}{b}\right)^2\left(\frac{F_1 - F_2}{r_1 - r_2}\right)$ (vii)

If the weight of the sleeve is W, it should be taken into account and it must be added to the spring force S when obtaining the relation between F and S.

If the friction force R at the sleeve is considered, it is added to or subtracted from the spring force S according as the speed is increasing or decreasing, while obtaining the relation F and S.

The initial compression of the spring is obtained by the equation

δ = Initial compression of the spring $= \frac{S_1}{k}$ (viii)

Example (4)

A spring loaded controlled governor has two balls each weighing 2 kg and each attached to the arm of a bell crank lever which pivots about a fixed fulcrum. The other arms of the bell crank levers carry rollers, which lift the sleeve against the pressure exerted by a spring surrounding the governor spindle. The two arms of the each bell crank lever are of equal length and the minimum and maximum radii of rotation of the governor balls are 7.5 cm and 12 cm respectively. If the sleeve is to begin to lift at 240 rpm and the increase of speed allowed is 7%, find the initial load on the sleeve and the required stiffness of the spring. Refer to fig-3.10.

As the two arms of the bell crank lever are equal, the centrifugal force on each ball at any instance is equal to half the spring force at that instant.

At the lowest position, the radius of rotation of the ball path is 7.5 cm and the speed of rotation is 240 rpm.

$$\omega_2 = \frac{240 \times 2\pi}{60} = 25.13 \text{ rad/s}$$

$$F_2 = \frac{w}{g}\omega_2^2 r_2 = \frac{2}{981} \times (25.12)^2 \times 7.5 = 9.65 \text{ kg}$$

$S_2 = 2 \times 9.65 = 19.3$ kg.

At the highest position, the radius of rotation of the ball path is 12 cm and the speed is $1.07 \times 240 = 256.8$ rpm.

$$\omega_2 = \frac{256.8 \times 2\pi}{60} = 26.89 \text{ rad/s}$$

$$F_1 = \frac{w}{g}\omega_1{}^2 r_1 = \frac{2}{981} \times (26.89)^2 \times 12 = 17.69 \text{ kg}$$

$S_1 = 2 \times 17.69 = 35.38$ kg.

As the two arms of the bell crank lever are equal, the lift of the sleeve is equal to the change in radius, which is equal to $12 - 7.5 = 4.5$ cm.

$\therefore$ Stiffness of the spring $= \dfrac{S_1 - S_2}{4.5} = \dfrac{35.38 - 19.30}{4.5} = 3.57$ kg/cm.

Example (5)

A Hartnell governor has two rotating balls, weighing 2.7 kg each. The ball radius is 12.5 cm in the mean position when the ball arms are vertical and the speed is 150 rpm with the sleeve rising. The length of the ball arm is 15 cm and the sleeve arm is 9 cm. The stiffness of the spring is 8 kg/cm and the total sleeve movement is ± 1.5 cm from the mean position. Allowing for a constant friction force of 1.5 kg acting at the sleeve, determine the highest and lowest speed of the governor.

Mean speed is 150 rpm. $\therefore \omega = \dfrac{150 \times 2\pi}{60} = 15.7$ rad/s

F = centrifugal force on the ball at the mean position

$$= \frac{w}{g}\omega^2 r = \frac{2.7}{981} \times (15.7)^2 \times 12.5 = 8.5 \text{ kg}$$

If S be the spring force in the mean position, with the sleeve rising, then

$$\frac{(S + 1.5)}{2} \times 9 = 8.5 \times 15$$

$$S = 26.8 \text{ kg}$$

The lift of the sleeve from mid position is 1.5 cm either side. As the stiffness of the spring is 8 kg/cm, the change in spring force is $8 \times 1.5 = 12$ kg from the spring force in mid position.

$\therefore S_1 = 26.8 + 12 = 38.8$ kg.

$S_2 = 26.8 - 12 = 14.8$ kg

The change in radius of the ball path from mid position is $\frac{15\times1.5}{9} = 2.5$ cm.

$\therefore \quad r_1 = 12.5 + 2.5 = 15$ cm

$r_2 = 12.5 - 2.5 = 10$ cm.

The maximum speed occurs at the top of sleeve position with the sleeve rising. At maximum radius position, take moment about fulcrum point.

$$F_1 \times 15 = \frac{38.8+1.5}{2} \times 9$$

$$F_1 = 12.1 \text{ kg}$$

If ω_1 be the speed of the governor spindle in rad/s, then

$$12.1 = \frac{2.7}{981} \times 15 \times \omega_1^{\,2}$$

$$\omega_1 = 17.11 \text{ rad/s} \qquad \text{(or)} \qquad N_1 = \frac{17.11 \times 60}{2\pi} = 163.4 \text{ rpm}$$

When the sleeve is falling to the bottom of the sleeve, the speed of the spindle is minimum at bottom. At minimum radius, take moment about fulcrum point.

$$F_2 \times 15 = \frac{14.8-1.5}{2} \times 9$$

$$F_2 = 4 \text{ kg.}$$

If ω_2 be the speed of the governor spindle in rad/s, then

$$4 = \frac{2.7}{981} \times 10 \times \omega_2^{\,2}$$

$$\omega_2 = 12.04 \text{ rad/s} \qquad \text{(or)} \qquad N_2 = \frac{12.04 \times 60}{2\pi} = 115 \text{ rpm.}$$

(b) Spring Loaded Governors (Balls Directly Connected by Springs)

Fig-3.11 (a) shows a governor in which the balls are connected by two springs in tension. In fig-3.11 (a) the governor spindle is vertical and the path of the centers of the ball path is in a horizontal plane.

It is assumed that the two springs are identical and the effect of gravity on the sleeve is neglected. Let S be the sum of the tensions in the two springs and F be the centrifugal force when the radius of rotation of the ball path is r. From equilibrium of forces,

$$S = F \qquad \text{(i)}$$

If k is the stiffness of each spring and if the tension in each spring increases from $\frac{S_2}{2}$ to $\frac{S_1}{2}$ as the spring length increases from $2r_2$ to $2r_1$, then

$$k = \frac{\frac{(S_1 - S_2)}{2}}{2(r_1 - r_2)} = \frac{(S_1 - S_2)}{4(r_1 - r_2)} \qquad \text{(ii)}$$

The initial extension of the spring, when the governor floats, is

$$\delta = \frac{\frac{S_2}{2}}{k} = \frac{S_2}{2k} \qquad \text{(iii)}$$

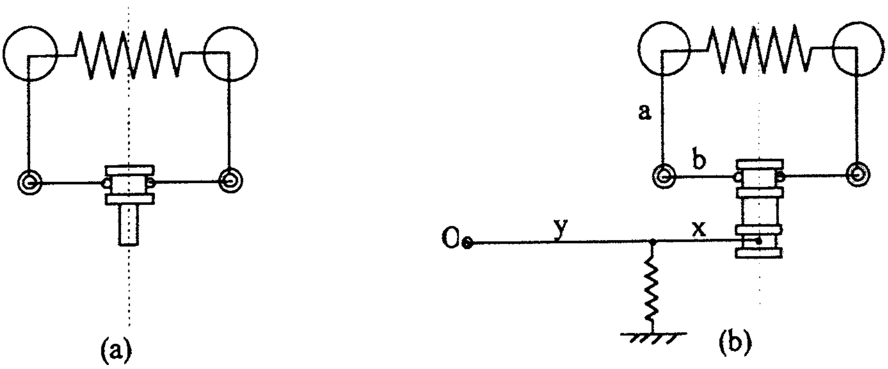


Fig-3.11

Since the spring force in this type of governor cannot be adjusted it is usual to have a supplementary adjustable spring, which acts on a lever and provides a force on the sleeve as shown in fig-3.11 (b). In this case the spring force S is less than the centrifugal force and the relation between the forces can be obtained by taking moment about the fulcrum of the bell crank lever.

Example (6)

In a spring loaded controlled governor as shown in fig-3.11 (a), the two operating masses weighing 1 kg each rotate in a cycle of 20 cm diameter when in mid position. The two controlling springs, which connected directly the two operating masses in parallel have a stiffness of 0.9 kg/cm and are extended 6.5 cm when the governor is in mid position. Determine the equilibrium speeds of the governor for mid position and for the case when the balls rotate in a cycle of 23 cm diameter.

In mid position, the radius of rotation of the ball path is 10 cm. The spring force acting on the ball is 2 × 0.9 × 6.5 = 11.7 kg. The centrifugal force on the ball will be 11.7 kg. If ω be the speed of rotation of the governor spindle in rad/s, then

$$\frac{1}{981}\times 10\times \omega^2 = 11.7$$

$$\omega = 34 \text{ rad/s.}$$

$$N = \frac{34\times 60}{2\pi} = 324 \text{ rpm}$$

When the radius of rotation of the ball path is $\frac{23}{2} = 11.5$ cm, the total extension of each spring will be 6.5 + (23 – 20) = 9.5 cm. Therefore, the spring force acting on each ball in new configuration will be 2 × 0.9 × 9.5 = 17.1 kg. The centrifugal force will be also equal to 17.1 kg. If ω' be the speed of rotation of the governor spindle, in rad/s, in new configuration, then

$$\frac{1}{981}\times 11.5\times \omega'^2 = 17.1$$

$$\omega' = 37.2 \text{ rad/s}$$

$$N' = \frac{37.2\times 60}{2\pi} = 364 \text{ rpm}$$

Example (7)

In a spring loaded controlled governor is shown in fig-3.11 (b). The two balls each of weight 5.45 kg are connected across by two springs 'A'. A supplementary spring 'B' provides an additional force at the sleeve through a medium of a lever, which pivots about a fixed center at its left hand end. In the given position, the radius of the governor ball is 15 cm and the speed is 600 rpm. The tension in each spring 'A' is then 110 kg. Find the tension in the spring 'B' for this position. If, when the sleeve moves up 2 cm, the speed is 630 rpm, find the necessary stiffness of the spring 'B', if the stiffness of each spring 'A' is 8 kg/cm. Take a = b = 11 cm, x = 10 cm and y = 20 cm.

When the radius of rotation of the ball path is 15 cm, the equilibrium speed is 600 rpm. Therefore the centrifugal force on the ball is

$$\frac{5.45}{981} \times 15 \times \left(\frac{600 \times 2\pi}{60}\right)^2 = 329 \text{ kg.}$$

The total spring force = 2 × 110 = 220 kg.

If W were the force on the sleeve for this configuration, then by taking moment about the fulcrum of the bell crank lever,

$$\frac{W}{2} \times 11 = (329 - 220) \times 11$$

$$\therefore \quad W = 2\,(329 - 220) = 218 \text{ kg.}$$

If S be the tension in the spring 'B' for this configuration, then by taking moment about the fixed point O,

$$S \times 20 = 218 \times 30$$

$$S = \frac{216 \times 30}{20} = 327 \text{ kg}$$

When the equilibrium speed is 630 rpm,

The radius of rotation of the ball path will be 15 + 2 = 17 cm, because the lift of the sleeve is equal to change in radius because two arms of the bell crank levers are equal. The extension of each spring 'A' is 2 × 2 = 4 cm. The tension in each spring 'A' in new configuration will be 110 + 4 × 8 = 142 kg. The total spring force acting radially on each ball will be 2× 142 = 248 kg. The centrifugal force acting radially outwards on each ball in new configuration is

$$\frac{5.45}{981} \times 17 \times \left(\frac{630 \times 2\pi}{60}\right)^2 = 411 \text{ kg.}$$

If W_1 is the force on the sleeve for the new configuration, then by taking moment about the fulcrum of the bell crank lever,

$$\frac{W_1}{2} \times 11 = (411 - 284) \times 11$$

$$W_1 = 254 \text{ kg.}$$

If S_1 is the tension in the spring 'B' for this configuration, then

$$S_1 \times 20 = 254 \times 30$$

$$S_1 = \frac{250 \times 30}{20} = 381 \text{ kg.}$$

The extension of the spring, for 2 cm movement of the sleeve is cm. $2\times\frac{2}{3}=\frac{4}{3}$ cm

$$\therefore \text{Stiffness of spring B} = \frac{(381-327)}{\frac{4}{3}} = 40.5 \text{ kg/cm.}$$

(c) Governor with Spring and Gravity Loaded Controlled

Another type of spring-loaded governor is shown in fig-3.12 (a), in which the movement of the sleeve is controlled by a compression spring together with a dead load. The bell crank lever L are carried on pivots O which are attached to the sleeve, which is free to slide relative to the governor spindle, which has a cap or a disc fixed to its upper end. The spring is compressed between the sleeve and the cap. The rollers R on the ends of the horizontal arms of the bell crank levers press on the cap C so that the sleeve is lifted against the compression of the spring.

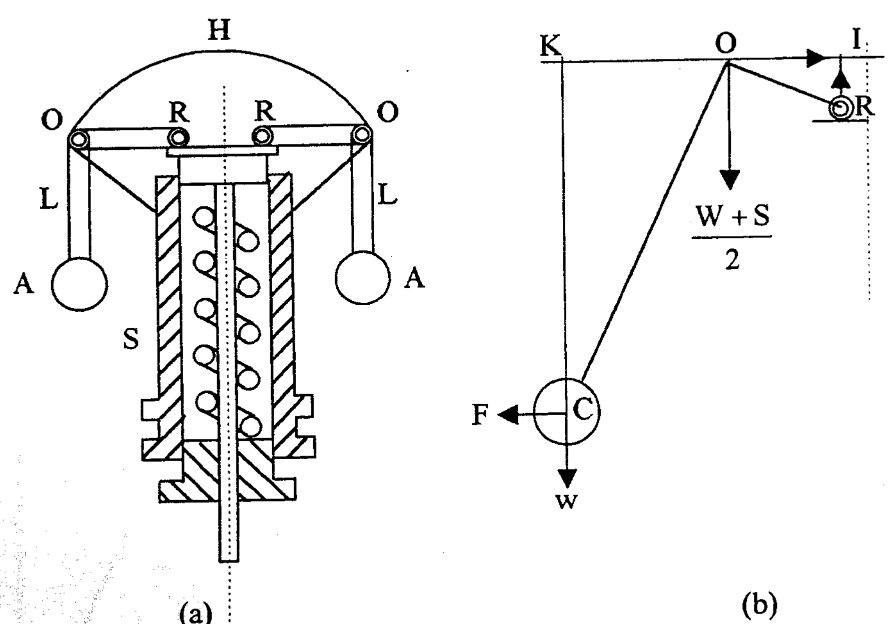


Fig-3.12

Fih-3.12 (b) shows forces acting on one bell crank lever when the governor is running at an equilibrium speed.

let W be the weight of the sleeve, S the force due to spring compression, F the centrifugal force acting on the ball and w the weight of the ball.

At the ball center, there are two forces, w and F. At the pivot O, there is a vertical force $\frac{W+S}{2}$ and a horizontal reaction, whose magnitude is not known. There is a vertical reaction at the roller R.

This type of governor may be analysed the simple manner by taking moments, about the instantaneous center of the bell crank lever of all the forces, which act on one of the bell crank levers.

Since the roller moves horizontally and the path of the fulcrum is parallel to the governor axis, the instantaneous center I of the bell crank lever is the line through O at right angles to the governor axis.

Taking moment about I,

$$F \times CK = w \times KI + \frac{W+S}{2} \times OI \qquad \text{(i)}$$

When the various dimensions are known, the problem can be solved.

Example (8)

In the governor shown in fig-3.12 (a), the weight of the sleeve is 12 kg and of each ball is 2.4 kg and the minimum radius of rotation of the balls is 10 cm, find the initial compression in the spring and also the stiffness of the spring in order that the sleeve shall begin to rise at 240 rpm and rise to 0.63 cm when the speed increases to 265 rpm. The length of the ball arm is 15 cm and the roller arm of 7.5 cm.

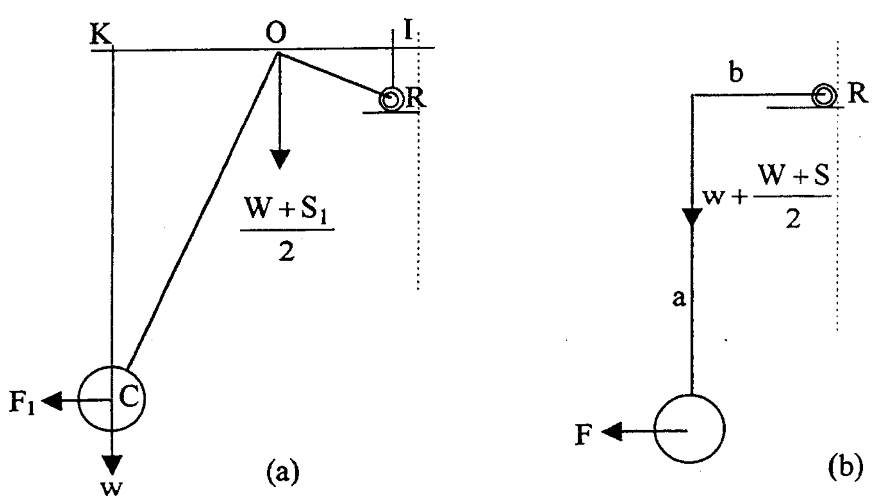


Fig-3.13

Fig-3.13 (a) and (b) show the configuration of the governor and the forces acting on it for maximum speed and minimum speed respectively. At minimum speed condition, the instantaneous center of the bell crank lever coincides with the center of the roller, R.

When the radius of rotation is 10 cm, the equilibrium speed is 240 rpm.

$$\therefore F = \frac{2.4}{981} \times 10 \times \left(\frac{240 \times 2\pi}{60}\right)^2 = 15.4\,kg$$

Taking moment about the instantaneous center at minimum speed condition,

$$15.4 \times 15 = (2.4 + \frac{12+S}{2}) \times 7.5$$

$$S = 45\ kg.$$

When the sleeve rises by 0.63 cm, the new radius of the ball path will be 10 + 2 × 0.63 = 11.26 cm, the new speed of rotation is 265 rpm.

$$\therefore F_1 = \frac{2.4}{981} \times 11.26 \times \left(\frac{265 \times 2\pi}{60}\right)^2 = 21.25\,kg.$$

$$OI = \sqrt{7.5^2 - 0.63^2} = 7.47\,cm$$

$$KI = 7.47 + (2 \times 0.63\ ) = 8.73\ cm.$$

$$CK = \sqrt{15^2 - (2 \times 0.63)^2} = 14.95\,cm$$

By taking moment about I, the instantaneous center, which is shown in fig-3.13 (a)

$$21.25 \times 14.95 = 2.4 \times 8.73 + \frac{12+S_1}{2}$$

$$S_1 = 67.4\ kg.$$

$$\text{Stiffness of the spring} = k = \frac{67.5 - 45}{0.63} = 35.4\ kg/cm.$$

$$\text{Initial compression} = \delta = \frac{45}{35.4} = 1.27\ cm.$$

PROBLEMS

1. A Porter governor has four equal links of 18 cm long, and the points of attachment of the links are at a radial distance of 4 cm from the vertical axis of the governor. The two balls each weigh 1 kg, and the central weight including the sleeve weighs 20 kg. When the links are at angle of 30° to the vertical the governor sleeve begins to rise when the speed is 283 rpm. Estimate the higher and lower speeds of the governor when the angle of the link is 45° to the vertical, assuming that the friction of the governor remains constant.

2. Derive an expression for the height of a Porter governor with a central load.

In such a governor the upper and lower arms are each 25 cm long and they are attached on points on the governor spindle. On bottom position of the sleeve, the arms make an angle of 35° with the vertical spindle and in top position the angle of 45°. Assuming that the speed of the governor for mid position of the sleeve is the arimatical means of the speeds for top and bottom positions of the sleeve, find the ratio of the range of speed to the mean of speed. If the weight of each rotating ball is one eighth of the weight of the central load, fin the mean speed of the governor.

3. In the Proell governor, similar to one shown in fig-3.9 (a), the arm ABC is formed in one piece. Each ball weighs 2 kg and the effect of friction is equivalent to a force of 1 kg at the sleeve.

AB = 5 cm; BD = BC = 15 cm. The links and arms are attached to pins, which are at a distance of 2.5 cm from the vertical axis of the spindle as shown. For the configuration shown, CD = 22.5 cm. Find the magnitude of the central load on the sleeve, if the governor is to rise to its mean position as shown in figure, when the speed is 120 rpm, the portion AB of the lower arm then being vertical.

4. In a Hartnell governor the length of the ball arm is 17 8 cm, that of sleeve arm is 14 cm and the weight of each ball 2.27 kg. The ball radius with the arm vertical is 16.5 cm and the speed is 240 rpm. The speed is increased by 0.5% for a lift of 1.3 cm.

(a) Neglecting the dead load on the sleeve, find the necessary stiffness of the spring and the required initial compression.

(b) If the stiffness of the spring is increased by 10%, the speed with the ball arm vertical remaining the same, find the percentage increase in speed for the lift of 1.3 cm.

5. A spring loaded controlled governor of the Hartnell type is required to run at equilibrium speeds of 297 and 303 rpm, when the radius of rotation of the ball is 12.7 cm and 20.3 cm respectively. The length of the ball and the sleeve arms are 14 cm and 11.4 cm respectively. The weight of each ball is 4 kg. Find the mean change in the force on the spring per cm of its compression between the given limits.

The radius of the balls in mid position is 16.5 cm. Also, allowing 2% extra for frictional resistance, find the speeds when (a) the sleeve is rising, (b) the sleeve is falling.

6. In a spring-loaded governor, the two balls are connected by two identical helical springs and each ball weighs 6.6 kg. The arms are of equal length. An auxiliary spring is provided to obtain an additional downward force at the sleeve through a lever. This lever has a length of 37.5 cm and is pivoted at a point 25 cm from the governor sleeve.

The speed range is 425 to440 rpm with a sleeve movement of 0.635 cm. The minimum ball path is 12.4 cm. If the combined strength of the ball springs is 15.4 kg/cm, find the stiffness of the auxiliary spring. If, with the auxiliary removed, the speed at the maximum radius is 418 rpm, find the unstretched length of the ball springs.

7. In a vertical spring-loaded, the bell crank levers are pivoted at 15.24 cm radius. The ball arms are vertical and 15.24 cm long. The horizontal arms are 7.62 cm long. Each ball has an effective weight of 3.63 kg. The balls are connected by two identical helical springs and the motion of the governor sleeve is transmitted through a lever to a spring S, which has a stiffness of 14.29 kg/cm of elongation. The length of this lever is 40 cm and it is pivoted at a point 17.78 cm from the governor sleeve. The governor has a normal speed of 300 rpm, the rise in speed at no load is 3% and the sleeve moves 3.175 cm to cut off steam. Determine the necessary stiffness of each ball springs and find what additional extension of spring S will be required to raise the normal speed by 5%.

8. In a governor similar to one shown in fig-3.12 (a), the sleeve which weighs 9.08 kg is keyed to the spindle and is capable of axial movement along the spindle, which is vertical. Two rotating masses each of weight 2.27 kg are attached to bell crank levers pivoted on the lugs of the sleeve. The minimum radius of rotation of the mass centers of the balls is 8.9 cm. Find the initial compression of the central spring and the spring stiffness, in order that the sleeve shall begin to rise at 300 rpm, reaching a sleeve lift of 0.952 cm when the speed increases to 330 rpm. The ball arm is 12.7 cm and the roller arm is 6.35 cm long.

9. In a governor of the type shown in fig-3.12 (a), the weight o each ball is 1.362 kg, the weight of sleeve 4.54 kg, the length of the arms a and b of the bell crank levers 12.7 cm and 5.08 cm, the distance of the fulcrum from the axis of rotation and the minimum radius of rotation of the governor balls are each 6.35 cm. Find the initial thrust in the spring and the stiffness of the spring in order that the sleeve may begin to rise at 300 rpm and may rise 1.02 cm for in increase of speed of 5%.