UNIT 6 GOVERNORS

STRUCTURE

Compiled By

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6.1 Introduction

Flywheel which minimizes fluctuations of speed within the cycle but it cannot minimize fluctuations due to load variation. This means flywheel does not exercise any control over mean speed of the engine. To minimize fluctuations in the mean speed which may occur due to load variation, governor is used. The governor has no influence over cyclic speed fluctuations but it controls the mean speed over a long period during which load on the engine may vary.

The function of governor is to increase the supply of working fluid going to the prime-mover when the load on the prime-mover increases and to decrease the supply when the load decreases so as to keep the speed of the prime-mover almost constant at different loads.

Example: when the load on an engine increases, its speed decreases, therefore it becomes necessary to increase the supply of working fluid. On the other hand, when the load on the engine decreases, its speed increases and hence less working fluid isrequired.

When there is change in load, variation in speed also takes place then governor operates a regulatory control and adjusts the fuel supply to maintain the mean speed nearly constant. Therefore, the governor automatically regulates through linkages, the energy supply to the engine as demanded by variation of load so that the engine speed is maintained nearly constant.

6.1.1 Objectives

After studying this unit, you should be able to

- classify governors,
- analyse different type of governors,
- know characteristics of governors,
- know stability of spring controlled governors, and
- compare different type of governors.

6.2 Types of Governors

The governors may, broadly, be classified as

1. Centrifugal governors, and
2. Inertia governors.

The centrifugal governors, may further be classified (Fig. 1) as follows :

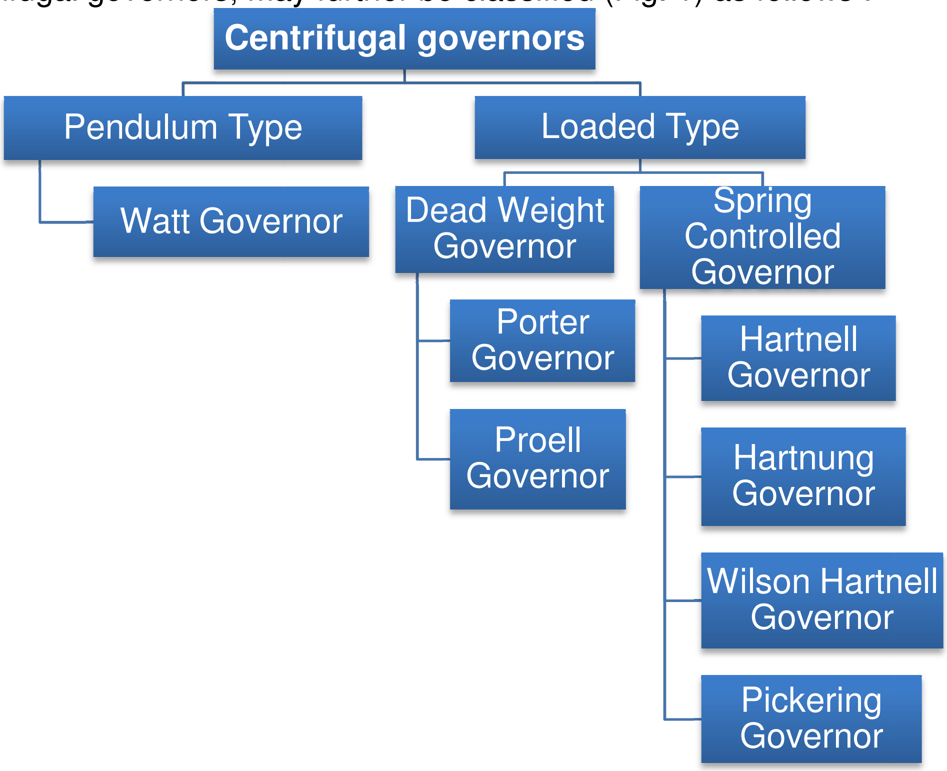


Fig.1 Classification of Centrifugal Governors

6.2.1 Centrifugal Governors

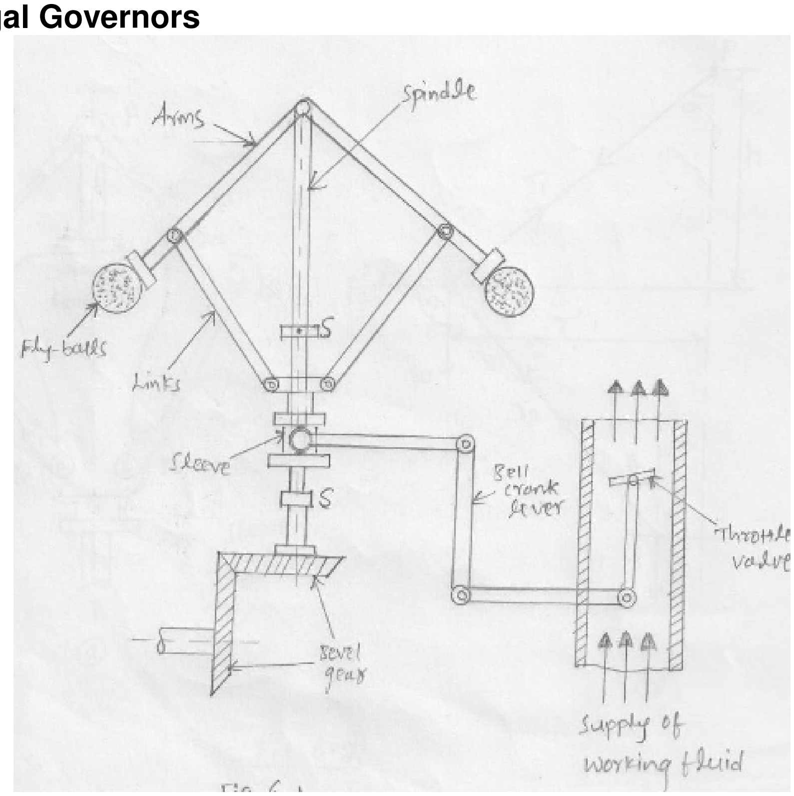


Fig.6.2 Schematic diagram of a centrifugal Governor

The centrifugal governors are based on the balancing of centrifugal force on the rotating balls by an equal and opposite radial force, known as the ***controlling force********.It consists of two balls of equal mass, which are attached to the arms as shown in Fig 6.2 These balls are known as ***governorballs* or *fly balls*.** The balls revolve with a spindle, which is driven by the engine through bevel gears. The upper ends of the arms are pivoted to the spindle, so that the balls may rise up or fall down as they revolve about the vertical axis. The sleeve revolves with the spindle; but can slide up and down. The balls and the sleeve rises when the spindle speed increases, and falls when the speed decreases. In order to limit the travel of the sleeve in upward and downward directions, two stops *S*, *S* are provided on the spindle. The sleeve is connected by a bell crank lever to a throttle valve. The supply of the working fluid decreases when the sleeve rises and increases when it falls. When the load on the engine increases, the engine and the governor speed decreases. This results in the decrease of centrifugal force on the balls. Hence the balls move inwards and the sleeve moves downwards. The downward movement of the sleeve operates a throttle valve at the other end of the bell crank lever to increase the supply of working fluid and thus the engine speed is increased. Hence, the extra power output is provided to balance the increased load. When the load on the engine decreases, the engine and the governor speed increases, which results in the increase of centrifugal force on the balls. Thus the balls move outwards and the sleeve rises upwards. This upward movement of the sleeve reduces the supply of the working fluid and hence the speed is decreased. Hence, the power output is reduced.

6.2.2 Inertia Governors

This works on a different principle. The governor balls are arranged so that the inertia forces caused by angular acceleration or retardation of the governor shaft tend to alter their positions. The amount of the displacement of the balls is controlled by springs and the governor mechanism to alter the supply of energy to the engine.

The advantage of this type of governor is that the positions of the balls are affected by the rate of change of speed of the governor shaft. Consequently, a more rapid response to a change of load is obtained, since the action of the governor is due to acceleration and not to a finite change of speed. The advantage is offset, however, by the practical difficulty of arranging for a complete balance of the revolving parts of the governor. For this reason centrifugal governors are much more frequently used.

6.3 Porter Governor and its Force analysis

The Porter governor is a modification of a Watt's governor, with central load attached to the sleeve as shown in Fig. 6.3(*a*). The load moves up and down the central spindle. This additional downward force increases the speed of revolution required to enable the balls to rise to any predetermined level.

Consider the forces acting on one-half of the governor as shown in Fig. 6.3 (*b*).

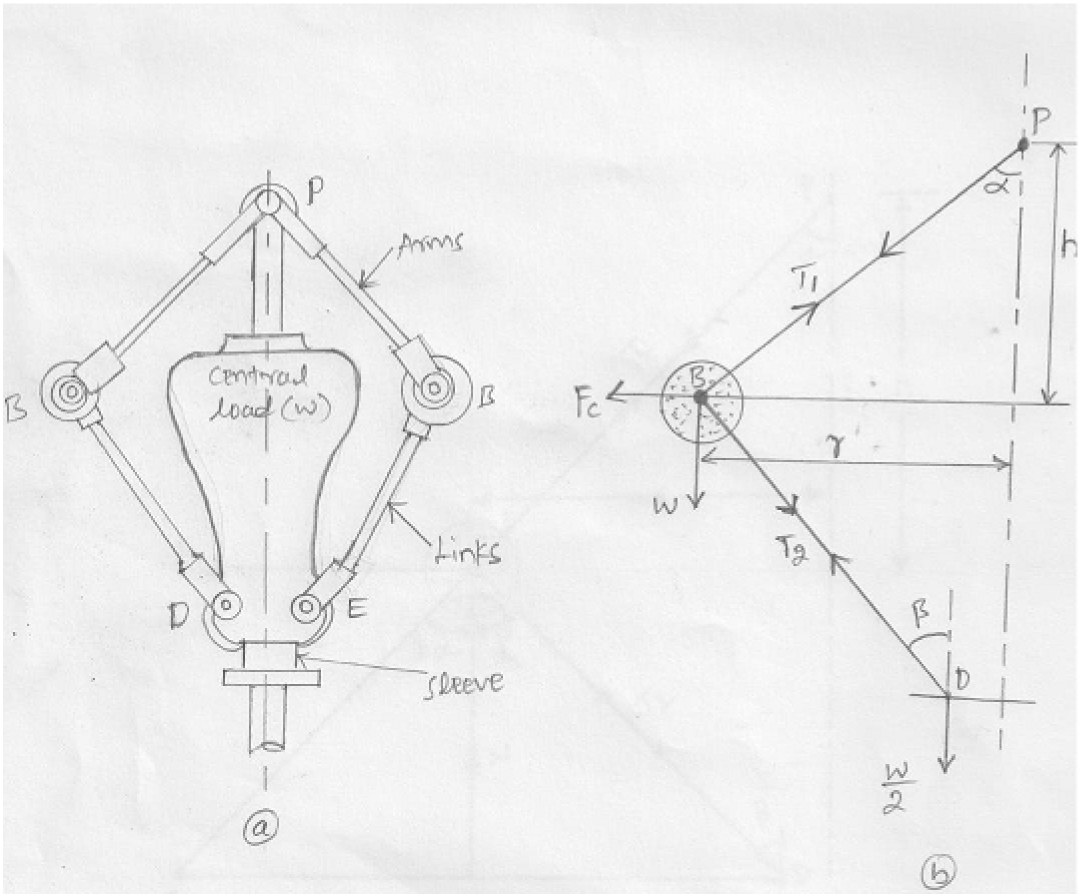


Fig 6.3 Porter governor.

m = Mass of each ball in kg,

w = Weight of each ball in N,

M = Mass of the central load in kg,

W = Weight of the central load in N,

r = Radius of rotation in m,

h = Height of governor in m,

N = Speed of the balls in rpm.

ω = Angular speed of the balls (2 Π*N*/60) rad/s,

*F*C = Centrifugal force acting on the ballN,

*T*1 = Force in the arm in N,

*T*2 = Force in the link in N,

α= Angle of inclination of the arm (orupper link) to the vertical, and

β = Angle of inclination of the link(or lower link) to the vertical.

Relation between the height of the governor (*h*) and theangular speed of balls(ω).

1. Method of resolution of forces.
2. Instantaneous centre method.

6.3.1 Method of resolution of forces

Considering the equilibrium of the forces acting at *D*, we have

$$T_2 cos\beta = \frac{W}{2} = \frac{M.g}{2}$$

$$T_2 = \frac{M.g}{2cos\beta} \tag{1}$$

Also considering the equilibrium of the forces acting on *B*the point *B* is in equilibrium under the action of the following forces, as shown in Fig. 6.3(*b*).

(*i*) The weight of ball (*w*),
(*ii*) The centrifugal force (*F*C),
(*iii*) The tension in the arm (*T*1), and
(*iv*) The tension in the link (*T*2).

Resolving the forces vertically

$$T_1 cos\alpha = T_2 cos\beta + w = \frac{M.g}{2} + m.g \tag{2}$$

Resolving the forces horizontally

$$T_1 sin\alpha + T_2 sin\beta = F_c$$

$$T_1 sin\alpha + \frac{M.g}{2cos\beta} * sin\beta = F_c$$

$$T_1 sin\alpha + \frac{M.g}{2} * tan\beta = F_c$$

$$T_1\, sin\alpha = F_c - \frac{M.g}{2} * tan\beta \tag{3}$$

Dividing equation (3) by equation (2)

$$\frac{T_1 sin\alpha}{T_1 cos\alpha} = \frac{F_c - \frac{M.g}{2} * tan\beta}{\frac{M.g}{2} + m.g}$$

$$\left(\frac{M.g}{2} + m.g\right)\tan a = F_c - \frac{M.g}{2} * tan\beta$$

$$\left(\frac{M.g}{2} + m.g\right) = \frac{F_c}{tan\alpha} - \frac{M.g}{2} * \frac{tan\beta}{tana}$$

Substituting $\frac{tan\beta}{tana} = q$ and $tana = \frac{r}{h}$ we have

$$\left(\frac{M.g}{2} + m.g\right) = \frac{h * F_c}{r} - \frac{M.g}{2} * q$$

$$\{F_c = m\omega^2 r\}$$

$$\left(\frac{M.g}{2} + m.g\right) = \frac{h * m\omega^2 r}{r} - \frac{M.g}{2} * q$$

$$h = \left[m.g + \frac{M.g}{2}(1+q)\right]\frac{1}{m\omega^2}$$

$$\omega^2 = \left[m.g + \frac{M.g}{2}(1+q)\right]\frac{1}{mh}$$

$$\left(\frac{2\pi N}{60}\right)^2 = \frac{m + \frac{M}{2}(1+q)}{m} * \frac{g}{h}$$

$$N^2 = \frac{m + \frac{M}{2}(1+q)}{m} * \frac{g}{h}\left(\frac{60}{2\pi}\right)^2$$

$$\{g = 9.81\}$$

$$N^2 = \frac{m + \frac{M}{2}(1+q)}{m} * \left(\frac{895}{h}\right)$$

When the length of arms are equal to the length of links and the points *P* and *D* lie on the same vertical line, then
$tana = tan\beta$ and q=1,

$$N^2 = \left(\frac{m + M}{m}\right) * \left(\frac{895}{h}\right) \qquad (4)$$

When the loaded sleeve moves up and down the spindle; the frictional force acts on it in a direction opposite to that of the motion of sleeve.

$$N^2 = \frac{mg + (\frac{Mg \pm F}{2})(1+q)}{mg} * \left(\frac{895}{h}\right)$$

$N^2 = \frac{mg+(Mg\pm F)}{mg} * \left(\frac{895}{h}\right)$ (for q=1)

The + sign is used when the sleeve moves upwards or the governor speed increases and - sign is used when the sleeve moves downwards or the governor speed decreases.

6.4 Instantaneous centre method

In this method, equilibrium of the forces acting on the link *BD* were considered. The instantaneous centre *I* lies at the point of intersection of *PB* produced and a line through *D* perpendicular to the spindle axis, as shown in Fig. 6.4. Taking moments about the point *I*,

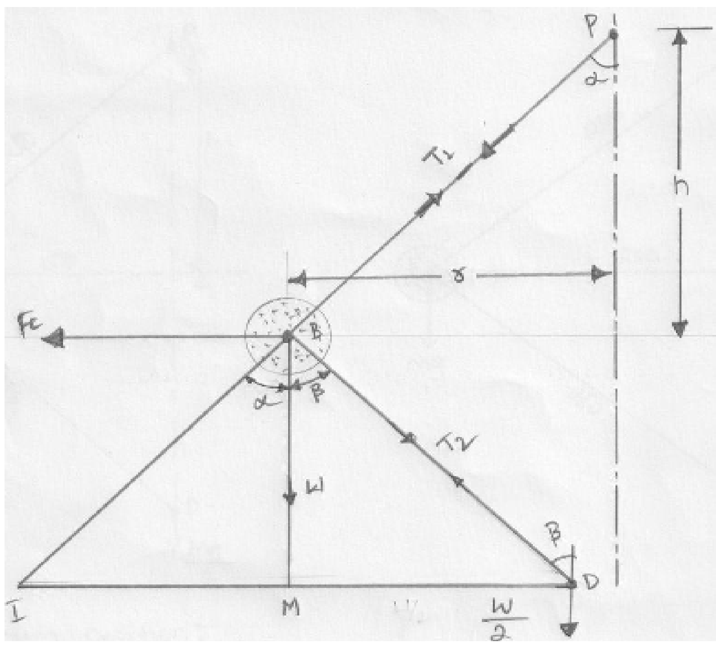


Fig 6.4 (Instantaneous centre method)

$$F_c * BM = w * IM + \frac{W}{2} * \mathrm{ID}$$

$$F_c = \mathrm{m} * \mathrm{g} * IM + \frac{M * g}{2} * \mathrm{ID}$$

$$F_c = \mathrm{m} * \mathrm{g} * \frac{IM}{BM} + \frac{M * g}{2} * \frac{\mathrm{ID}}{\mathrm{BM}}$$

$$F_c = \mathrm{m} * \mathrm{g} * \frac{IM}{BM} + \frac{M * g}{2} * \frac{\mathrm{IM} + \mathrm{MD}}{\mathrm{BM}}$$

$$F_c = \mathrm{m} * \mathrm{g} * \frac{IM}{BM} + \frac{M * g}{2} * \left(\frac{\mathrm{IM}}{\mathrm{BM}} + \frac{\mathrm{MD}}{BM}\right)$$

From Fig 6.4, $\frac{\mathrm{IM}}{\mathrm{BM}} = \tan\alpha, \frac{\mathrm{MD}}{BM} = tan\beta$

$$F_c = m * g * tan\alpha + \frac{M * g}{2} * (tan\alpha + tan\beta)$$

$$Dividing\ throughout\ by\ tan\alpha$$

$$\frac{F_c}{tan\alpha} = m * g + \frac{M * g}{2} * \left(1 + \frac{tan\beta}{tan\alpha}\right)$$

$\{q = \frac{tan\beta}{tan\alpha}\}$

We know that $F_c = m\omega^2 r$ and $tan\alpha = \frac{r}{h}$

$$m\omega^2 r * \frac{h}{r} = \mathrm{m} * \mathrm{g} + \frac{M * g}{2} * (1 + \mathrm{q})$$

$$h = \frac{m + \frac{M}{2}(1 + q)}{m} * \frac{g}{\omega^2}$$

when $tan\alpha = tan\beta\ or\ q = 1\ then$

$$\boldsymbol{h = \frac{m + M}{m} * \frac{g}{\omega^2}}$$

Porter Governor Problems

Problem 6.1

A Porter governor has equal arms each 250 mm long and pivoted on the axis of rotation. Each ball has a mass of 5 kg and the mass of the central load on the sleeve is 15 kg. The radius of rotation of the ball is 150 mm when the governor begins to lift and 200 mm when the governor is at maximum speed. Find the minimum and maximum speeds and range of speed of the governor.

Solution.

Given :*BP* = *BD* = 250 mm = 0.25 m ; *m* = 5 kg ; *M* = 15 kg ; *r*1 = 150 mm = 0.15m; *r*2 = 200 mm = 0.2 m

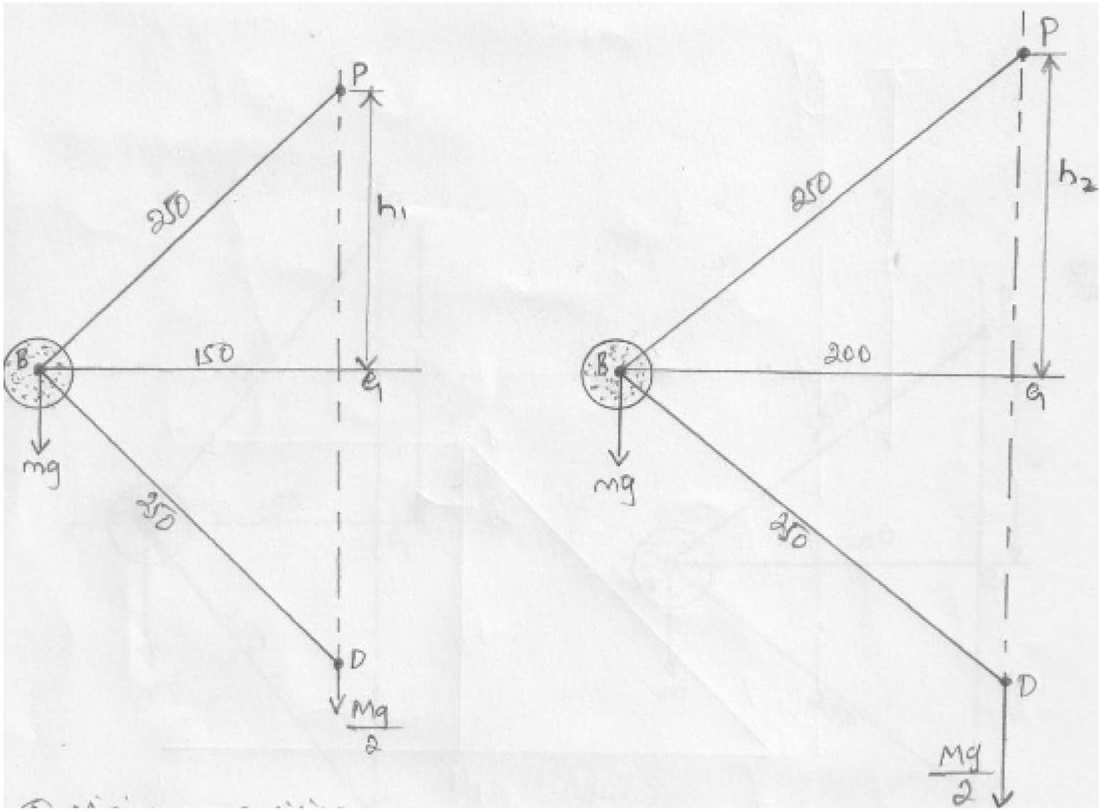


Fig 6p.1

The minimum and maximum positions of the governor are shown in Fig. 6p.1 (*a*) and (*b*) respectively.

Minimum speed

when r_1 = BG = 0.15 m

N1 = Minimum speed

Referring Fig. 6p.1(a), height of the governor,

,

$$h_1 = PG = \sqrt{(\mathrm{PB})^2 - (\mathrm{BG})^2} = \sqrt{(0.25)^2 - (0.15)^2} = \mathbf{0.2m}$$

Minimum speed of the governor is given by

$$N_1{}^2 = \left(\frac{m+M}{m}\right) * \left(\frac{895}{h_1}\right) = \left(\frac{5+15}{5}\right) * \left(\frac{895}{0.2}\right) = 17900$$

$$\boldsymbol{N_1 = 133.8\, rpm}$$

Maximum speed when r_2 = BG = 0.2 m

Let

$N2 = Maximumspeed.$

Referring Fig. 6p.1(b), height of the governor,

$$h_2 = PG = \sqrt{(\text{PB})^2 - (\text{BG})^2} = \sqrt{(0.25)^2 - (0.2)^2} = \mathbf{0.15m}$$

Maximum speed of the governor is given by

$${N_2}^2 = \left(\frac{m+M}{m}\right) * \left(\frac{895}{h_2}\right) = \left(\frac{5+15}{5}\right) * \left(\frac{895}{0.15}\right) = 23867$$

$$\boldsymbol{N_2 = 154.5\, rpm}$$

Range of speed

$$= \boldsymbol{N2 - N1 = 154.4 - 133.8 = 20.7\, rpm.}$$

Problem 6.2 The arms of a Porter governor are each 250 mm long and pivoted on the governor axis. The mass of each ball is 5 kg and the mass of the central sleeve is 30 kg. The radius of rotation of the balls is 150 mm when the sleeve begins to rise and reaches a value of 200 mm for maximum speed. Determine the speed range of the governor. If the friction at the sleeve is equivalent of 20 N of load at the sleeve, determine how the speed range is modified.

Solution;.Given : BP = BD = 250 mm ; m = 5 kg ; M = 30 kg ; r_1 = 150 mm ;
r_2 = 200 mm

Minimum and maximum speed of the governor

The minimum and maximum position of the governor is shown in Fig. 6p.2 (*a*) and (*b*) respectively.

*N*1 = Minimum speed when *r*1 = *BG* = 150 mm, and

*N*2 = Maximum speed when *r*2 = *BG* = 200 mm.

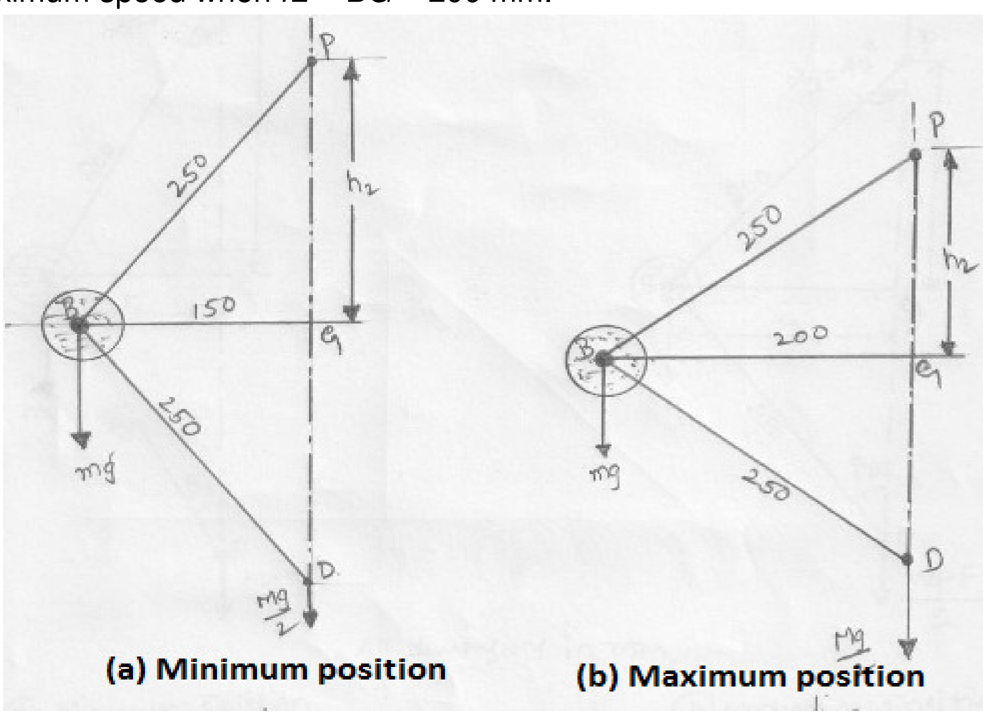


Fig 6p.2

To find Speed range of the governor

Referring Fig. 6p.2 (a), height of the governor,

$$h_1 = PG = \sqrt{(\mathrm{PB})^2 - (\mathrm{BG})^2} = \sqrt{(250)^2 - (150)^2} = 200mm = \mathbf{0.2m}$$

Minimumspeed of the governor is given by

$$N_1{}^2 = \left(\frac{m+M}{m}\right) * \left(\frac{895}{h_1}\right) = \left(\frac{5+30}{5}\right) * \left(\frac{895}{0.2}\right) = 31325$$

$$\boldsymbol{N_1 = 177\ rpm}$$

Referring fig 6p.2(b) height of the governor,

$$h_2 = PG = \sqrt{(\mathrm{PB})^2 - (\mathrm{BG})^2} = \sqrt{(250)^2 - (200)^2} = 150mm = \mathbf{0.15m}$$

Maximum speed of the governor is given by

$$N_2{}^2 = \left(\frac{m+M}{m}\right) * \left(\frac{895}{h_1}\right) = \left(\frac{5+30}{5}\right) * \left(\frac{895}{0.15}\right) = 41767$$

$$\boldsymbol{N_2 = 204.4\ rpm}$$

Speed range of the governor is given by

= N_2 – N_1 = 204.4 – 177 = 27.4 rpm.

Speed range when friction at the sleeve is equivalent of 20 N of load (i.e. when F = 20 N)

When the sleeve moves downwards, the friction force (*F*) acts upwards and the minimum speed isgiven by

$$N_1{}^2 = \left(\frac{m.g + (M.g - F)}{m.g}\right) * \left(\frac{895}{h_1}\right)$$

$$N_1{}^2 = \left(\frac{5 * 9.81 + (30 * 9.81 - 20)}{5 * 9.81}\right) * \left(\frac{895}{0.2}\right) = 29500$$

$$\boldsymbol{N_1 = 172\ rpm}$$

We also know that when the sleeve moves upwards, the frictional force (*F*) acts downwards and the maximum speed is given by

$$N_2{}^2 = \left(\frac{m.g + (M.g + F)}{m.g}\right) * \left(\frac{895}{h_2}\right)$$

$$N_2{}^2 = \left(\frac{5*9.81+(30*9.81+20)}{5*9.81}\right)*\left(\frac{895}{0.15}\right) = 44200$$

$$\boldsymbol{N_2 = 210\ rpm}$$

Speed range of the governor**= N_2 – N_1= 210 – 172 = 38 rpm.**

Problem 6.3 In an engine governor of the Porter type, the upper and lower arms are 200 mm and 250 mm respectively and pivoted on the axis of rotation. The mass of the central load is 15 kg, the mass of each ball is 2 kg and friction of the sleeve together with the resistance of the operating gear is equal to a load of 25 N at the sleeve. If the limiting inclinations of the upper arms to the vertical are 30° and 40°, find, taking friction into account, range of speed of the governor.

Solution.

Given :*BP* = 200 mm = 0.2 m ; *BD* = 250 mm = 0.25 m ; *M* = 15 kg ; *m* = 2 kg ;
F = 25 N ;α_1= 30°; α_2 = 40°

Minimum and maximum speed of the governor

The minimum and maximum position of the governor is shown Fig. 6p.3(*a*) and (*b*)respectively.

$N1$ = Minimum speed,

$N2$ = Maximum speed.

Referring Fig. 6p.3 (*a*), minimum speed,$r1$ = *BG* = *BP* sin 30° = 0.2 × 0.5 = 0.1 m

Height of the governor,$h1$ = *PG* = *BP* cos 30° = 0.2 × 0.866 = 0.1732 m

DG = $\sqrt{(BD)^2 - (BG)^2} = \sqrt{(0.25)^2 - (0.1)^2}$

DG = .23 m

$$\tan\beta_1 = \frac{BG}{DG} = \frac{0.1}{0.23} = 0.4348$$

$$\tan\alpha_1 = \tan 30° = 0.5774$$

$$q_1 = \frac{\tan\beta_1}{\tan\alpha_1} = \frac{0.4348}{0.5774} = 0.753$$

When the sleeve moves downwards, the frictional force (F) acts upwards and the minimum speed is given by

$$N_1{}^2 = \frac{mg + (\frac{Mg-F}{2})(1+q_1)}{mg} * \left(\frac{895}{h_1}\right)$$

$$N_1{}^2 = \frac{2*9.81 + (\frac{15*9.81-25}{2})(1+0.753)}{2*9.81} * \left(\frac{895}{0.1732}\right) = 33596$$

$$\boldsymbol{N_1 = 183.3\ rpm}$$

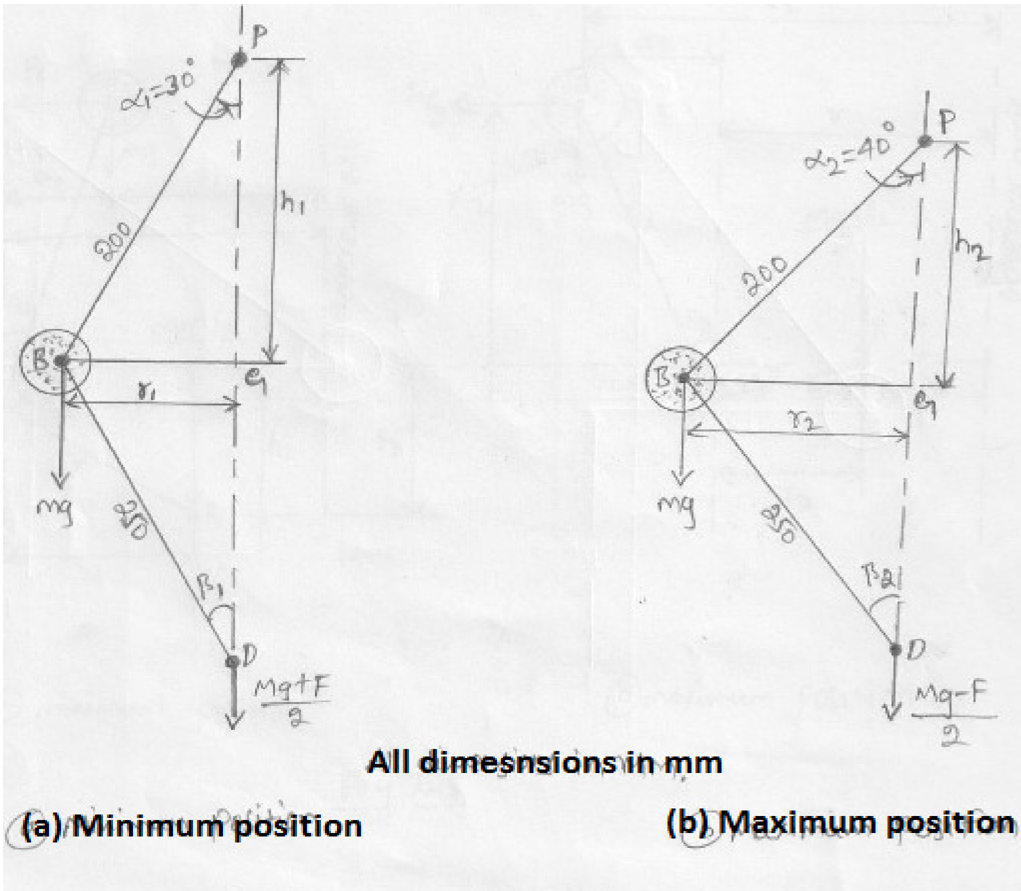


Fig 6p.3

Referring Fig. 6p.3(b),maximum speed,

$$r2 = BG = BPsin\,40° = 0.2 \times 0.643 = \mathbf{0.1268\,m}$$

Heightofthegovernor

$$h2 = PG = BPcos\,40° = 0.2 \times 0.766 = \mathbf{0.1532\,m}$$

$$DG = \sqrt{(BD)^2 - (BG)^2} = \sqrt{(0.25)^2 - (0.1268)^2}$$

DG = 0.2154 m

$$\tan\beta_2 = \frac{BG}{DG} = \frac{0.1268}{0.2154} = \mathbf{0.59}$$

$$\tan\alpha_2 = \tan 40° = \mathbf{0.839}$$

$$q_2 = \frac{\tan\beta_2}{\tan\alpha_2} = \frac{0.59}{0.839} = \mathbf{0.703}$$

When the sleeve moves upwards, the frictional force (*F*) acts downwards and the maximum speed is given by

$$N_2{}^2 = \frac{mg + (\frac{Mg+F}{2})(1+q_2)}{mg} * \left(\frac{895}{h_2}\right)$$

$$N_1{}^2 = \frac{2*9.81 + (\frac{15*9.81+25}{2})(1+0.753)}{2*9.81} * \left(\frac{895}{0.1532}\right) = 49236$$

$$\boldsymbol{N_2 = 222\ rpm.}$$

Range of speed

$$= \boldsymbol{N2 - N1 = 222 - 183.3 = 38.7\ rpm}$$

6.4 Hartnell Governor

A Hartnell governor is a spring loaded governor as shown in Fig. 6.4. It consists of two bell crank levers pivoted at the points O,O to the frame. The frame is attached to the governor spindle and therefore rotates with it. Each lever carries a ball at the end of the vertical arm OB and a roller at the end of the horizontal arm OR. A helical spring in compression provides equal downward forces on the two rollers through a collar on the sleeve. The spring force may be adjusted by screwing a nut up or down on the sleeve.

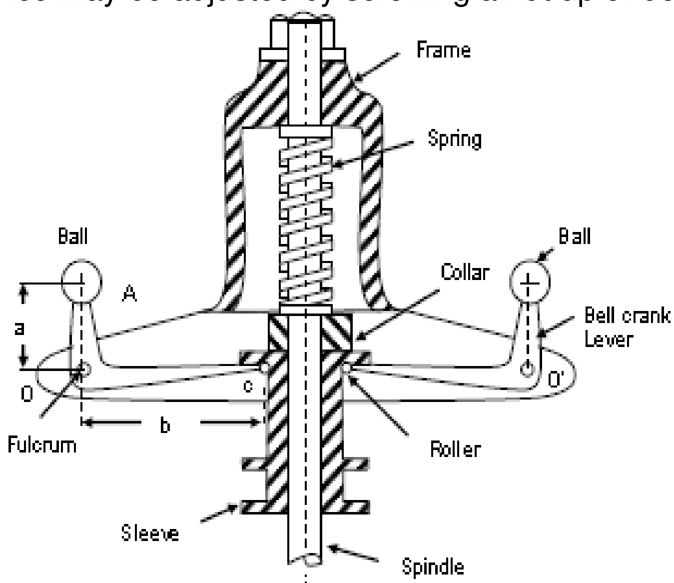


Fig. 6.5 Hartnell Governor

$Let\ m = Mass\ of\ each\ ball\ in\ kg,$

$M = Mass\ of\ sleeve\ in\ kg,$

$r1 = Minimum\ radius\ of\ rotation\ in\ m,$

$r2 = Maximum\ radius\ of\ rotation\ in\ m,$

$\omega 1 = Angular\ speed\ of\ the\ governor\ at\ minimum\ radius\ in\ rad/s,$

$\omega 2 = Angular\ speed\ of\ the\ governor\ at\ maximum\ radius\ in\ rad/s,$

$S1 = Spring\ force\ exerted\ on\ the\ sleeve\ at\ \omega 1\ in\ N,$

$S2 = Spring\ force\ exerted\ on\ the\ sleeve\ at\ \omega 2\ in\ N,$

$FC1 = Centrifugal\ force\ at\ \omega 1\ in\ N,$

$FC2 = Centrifugal\ force\ at\ \omega 2\ in\ N,$

$s = Stiffness\ of\ the\ spring\ (\frac{N}{mm})$

$x = Length\ of\ the\ vertical\ or\ ball\ arm\ of\ the\ lever\ in\ m,$

$y = Length\ of\ the\ horizontal\ or\ sleeve\ arm\ of\ the\ lever\ in\ m, and$

$r = Distance\ of\ fulcrum\ O\ from\ the\ governor\ axis\ or\ the\ radius\ of\ rotation$

$when\ the\ governor\ is\ in\ mid-position, in\ metres.$

Consider the forces acting at one bell crank lever. The minimum and maximum position is shown in Fig.6.5. Let *h* be the compression of the spring when the radius of rotation changes from $r1\ to\ r2$.

For the minimum position *i.e*. when the radius of rotation changes from $r\ to\ r1$, as shown in Fig. 6.5 (*a*), the compression of the spring or the lift of sleeve $h1$ is given by

$$\frac{h_1}{y} = \frac{a_1}{x} = \frac{r - r_1}{x} \qquad (1)$$

Similarly, for the maximum position *i.e*. when the radius of rotation changes from $r\ to\ r2$, as shown in Fig. 6.5 (*b*), the compression of the spring or lift of sleeve $h2$ is given by

$$\frac{h_2}{y} = \frac{a_2}{x} = \frac{r_2 - r}{x} \qquad (2)$$

Adding equation 1 and 2

$$\frac{h_1 + h_2}{y} = \frac{r_2 - r_1}{x}$$

$$\{h_1 + h_2 = h\}$$

$$h = (r_2 - r_1)\frac{y}{x} \qquad (3)$$

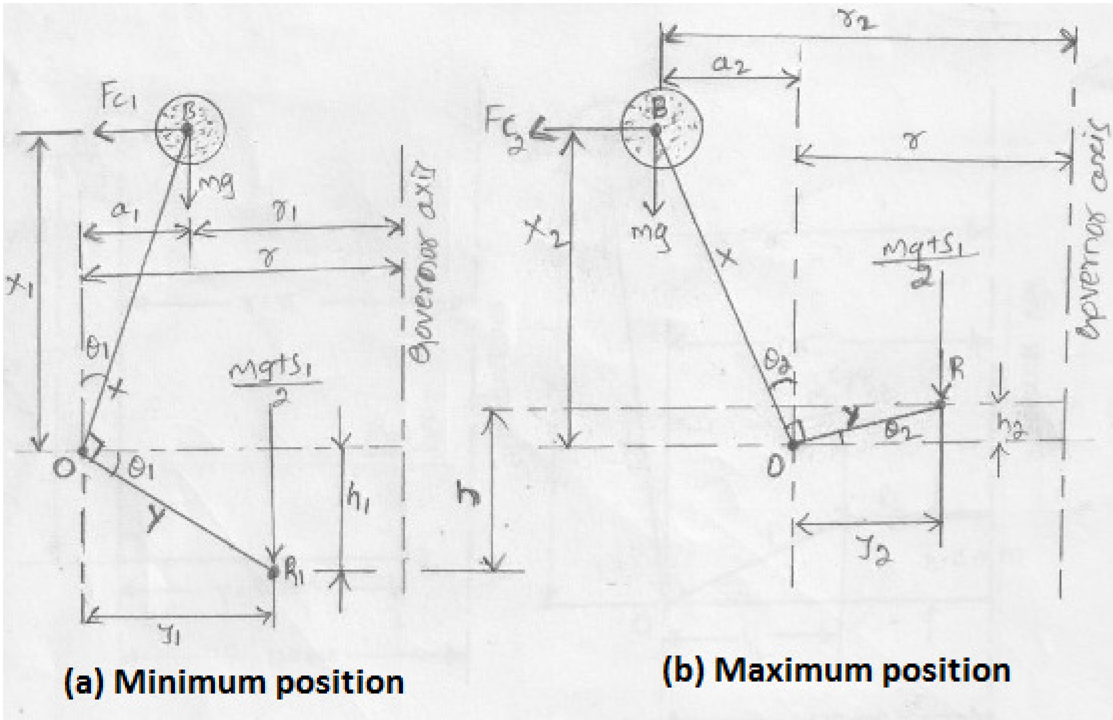


Fig 6.5 Hartnell Governor.

Now for minimum position, taking moments about point O, we get

$$\frac{M.g + s_1}{2} * y_1 = F_{c1} * x_1 - m.g * a_1$$

$$M.g + s_1 = \frac{2}{y_1}(F_{c1} * x_1 - m.g * a_1) \qquad (4)$$

Again for maximum position, taking moments about point O, we get

$$\frac{M.g + s_2}{2} * y_2 = F_{c2} * x_2 - m.g * a_2$$

$$M.g + s_2 = \frac{2}{y_2}(F_{c2} * x_2 + m.g * a_2) \qquad (5)$$

Subtracting equation (4) from equation (5),

$$s_2 - s_1 = \frac{2}{y_2}(F_{c2} * x_2 + m.g * a_2) - \frac{2}{y_1}(F_{c1} * x_1 - m.g * a_1)$$

We know that

$$s_2 - s_1 = h\, s \; and \; h = (r_2 - r_1)\frac{y}{x}$$

$$S = \frac{s_2 - s_1}{h} = \left(\frac{s_2 - s_1}{(r_2 - r_1)}\right)\frac{x}{y}$$

Neglecting the obliquity effect of the arms $(i.e. x_1 = x_2 = x, and\ y_1 = y_2 = y)$ and the moment due to weight of the balls $(i.e. mg)$,
we have for minimum position,

$$\frac{M.g + s_1}{2} * y = F_{c1} * x \qquad or \qquad M.g + s_1 = 2F_{c1} * \frac{x}{y} \tag{6}$$

Similarly for maximum position,

$$\frac{M.g + s_2}{2} * y = F_{c2} * x \qquad or \qquad M.g + s_2 = 2F_{c2} * \frac{x}{y} \tag{7}$$

Subtracting equation (6) from equation (7),

$$s_2 - s_1 = 2(F_{c2} - F_{c1})\frac{x}{y} \tag{8}$$

$$Also, s_2 - s_1 = h\,s \quad and \qquad h = (r_2 - r_1)\frac{y}{x}$$

$$s = \frac{s_2 - s_1}{h} = 2\left(\frac{F_{c2} - F_{c1}}{r_2 - r_1}\right)\left(\frac{x}{y}\right)^2 \tag{9}$$

Notes :

1. Unless otherwise stated, the obliquity effect of the arms and the moment due to the weight of the balls is neglected, in actual practice.

2. When friction is taken into account, the weight of the sleeve $(M.g)$ may be replaced by $(Mg \pm F)$

3. The centrifugal force (F_c) for any intermediate position (*i.e.* between the minimum and maximum position) at a radius of rotation (r) may be obtained as discussed below:

Since the stiffness for a given spring is constant for all positions, therefore for minimum and intermediate position,

$$s = 2\left(\frac{F_c - F_{c1}}{r - r_1}\right)\left(\frac{x}{y}\right)^2 \tag{10}$$

and for intermediate and maximum position,

$$s = 2\left(\frac{F_{c2} - F_c}{r_2 - r}\right)\left(\frac{x}{y}\right)^2 \ldots\ldots\ldots.(11)$$

From equation 9,10 and 11

$$\left(\frac{F_{c2} - F_{c1}}{r_2 - r_1}\right) = \left(\frac{F_c - F_{c1}}{r - r_1}\right) = \left(\frac{F_{c2} - F_c}{r_2 - r}\right)$$

$$F_c = F_{c1} + (F_{c2} - F_{c1})\left(\frac{r - r_1}{r_2 - r_1}\right) = F_{c2} - (F_{c2} - F_{c1})\left(\frac{r_2 - r}{r_2 - r_1}\right)$$

Problems on Hartnell Governor

Problem 6.4

A Hartnell governor having a central sleeve spring and two right-angled bell crank levers moves between 290 rpm. and 310 rpm. for a sleeve lift of 15 mm. The sleeve arms and the ball arms are 80 mm and 120 mm respectively. The levers are pivoted at 120 mm from the governor axis and mass of each ball is 2.5 kg. The ball arms are parallel to the governor axis at thelowest equilibrium speed. Determine

1. Loads on the spring at the lowest and the highest equilibriumspeeds, and

2. Stiffness of the spring.

Solution.

$Given: \ N1 = 290\, r.p.m. or \omega 1 = 2\pi \times \frac{290}{60} = 30.4 \frac{rad}{s};$

$N2 = 310\, r.p.m. or \omega 2 = 2\pi \times \frac{310}{60} = 32.5 \frac{rad}{s};$

$h = 15\, mm = 0.015\, m;$

$y = 80\, mm = 0.08\, m;$

$x = 120\, mm = 0.12\, m;$

$r = 120\, mm = 0.12\, m;$

$m = 2.5\, kg$

1. Loads on the spring at the lowest and highest equilibrium speeds

Let S = spring load at lowest equilibrium speed, and

S2 = spring load at highest equilibrium speed.

Since the ball arms are parallel to governor axis at the lowest equilibrium speed (*i.e.* at *N*1 = 290 rpm), as shown in Fig. 6p.4(*a*),

Therefore

$r = r1 = 120\, mm = 0.12\, m$

Centrifugal force at the minimum speed,

$F_{C1} = m(\omega_1)^2 r_1 = 2.5\,(30.4)^2 * 20.12 = 277\, N$

Now let us find the radius of rotation at the highest equilibrium speed, *i.e.* at *N*2 = 310 rpm. The position of ball arm and sleeve arm at the highest equilibrium speed is shown in Fig. 6p.4 (*b*).

Let $r_2 = Radius of rotation at N_2 = 310\ rpm$.
We know that

$$h = (r_2 - r_1)\frac{y}{x}$$

$$r_2 = r_1 + h\left(\frac{x}{y}\right) = 0.12 + 0.015\left(\frac{0.12}{0.08}\right) = \mathbf{0.1425m}$$

Centrifugal force at the maximum speed,

$$F_{c2} = m(\omega_2)^2 r_2 = 2.5\,(32.5)^2 * 0.1425 = \mathbf{376\ N}$$

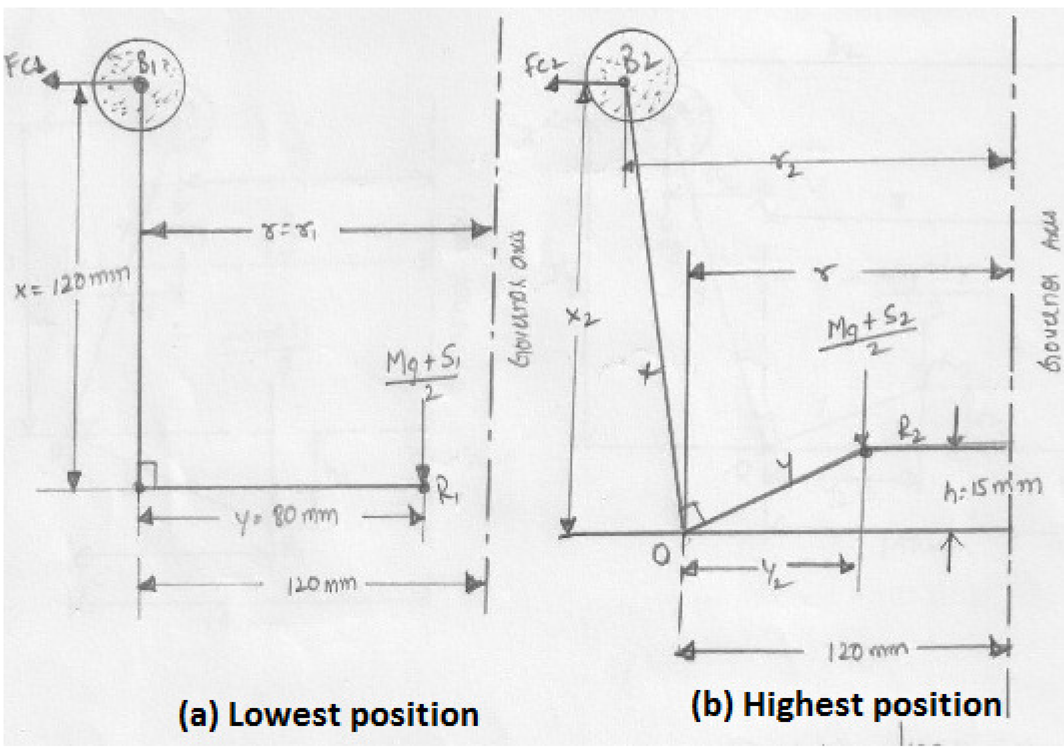


Fig 6p.4

Neglecting the obliquity effect of arms and the moment due to the weight of the balls, we have for lowest position,

$$M.g + S_1 = 2F_{c1} * \frac{x}{y} = 2 * 277 * \frac{0.12}{0.08} = \mathbf{831\ N}$$

$\{at\ M = 0\}$

$$\boldsymbol{S_1} = \mathbf{831\ N}$$

$$M.g + S_2 = 2F_{c2} * \frac{x}{y} = 2 * 376 * \frac{0.12}{0.08} = \mathbf{1128\ N}$$

$\{at\ M = 0\}$

$$\mathbf{S_2 = 1128\ N}$$

2. Stiffness of the spring
We know that stiffness of the spring,

$$s = \frac{S_2 - S_1}{h} = \frac{1128 - 831}{15} = \mathbf{19.8\ N/mm}$$

6.5
A spring loaded governor of the Hartnell type has arms of equal length. The masses rotate in a circle of 130 mm diameter when the sleeve is in the mid position and the ball arms are vertical. The equilibrium speed for this position is 450 r.p.m., neglecting friction. The maximum sleeve movement is to be 25 mm and the maximum variation of speed taking in account the friction to be 5 per cent of the mid position speed. The mass of the sleeve is 4 kg and the friction may be considered equivalent to 30 N at the sleeve. The power of the governor must be sufficient to overcome the friction by one per cent change of speed either way at mid-position. Determine, neglecting obliquity effect of arms;
1. The value of each rotating mass :
2. The spring stiffness in N/mm ; and
3. The initial compression of spring.

$\boldsymbol{Solution.} Given : x = y\ ;\ d = 130\ mm or r = 65\ mm = 0.065\ m\ ;$
$N = 450\ r.p.m. or$
$\omega = 2\pi \times \frac{450}{60} = 47.23 \frac{rad}{s};$
$h = 25\ mm = 0.025\ m\ ;$
$M = 4\ kg\ ;$
$F = 30\ N$

1. Value of each rotating mass
Let *m* = Value of each rotating mass in kg, and
S = Spring force on the sleeve at mid position in N.

Since the change of speed at mid position to overcome friction is 1 per cent either way (*i.e.* ± 1%), therefore
Minimum speed at mid position,

$$\omega = \omega - 0.01\omega = 0.99\omega = 0.99 \times 47.13 = \mathbf{46.66\ rad/s}$$

and maximum speed at mid-position

$\omega 2 = \omega + 0.01\omega = 1.01\omega = 1.01 \times 47.13 = \mathbf{47.6\,rad/s}$

Centrifugal force at the minimum speed,
$FC1 = m\,(\omega\ \ 1)^2 r = m\,(46.66)^2\,0.065 = \mathbf{141.5\,m\,N}$

and centrifugal force at the maximum speed,
$FC2 = m(\omega 2)^2 r = m(47.6)^2\,0.0065 = \mathbf{147.3\,mN}$

We know that for minimum speed at midposition,

$$S \quad + (M.g + F) = 2F_{c1} * \frac{x}{y}$$

$$S \quad + (4 * 9.81 - 30) = 2 * 141.5\text{m} * 1$$

$\{x = y\}$

$$S \quad + 9.24 = \mathbf{283\ m} \ldots \ldots \ldots \ldots (1)$$

and for maximum speed at mid-position,

$$S \quad + (M.g + F) = 2F_{c2} * \frac{x}{y}$$

$$S \quad + (4 * 9.81 + 30) = 2 * 147.3\text{ m} * 1$$

$\{x = y\}$

$$S \quad + 69.24 = \mathbf{294.6\ m} \ldots \ldots \ldots \ldots (2)$$

From equation (1) and (2)

$$\mathbf{m = 5.2kg}$$

2. Spring stiffness in N/mm
Let s = spring stiffness in N/mm.
Since the maximum variation of speed, considering friction is ± 5% of the mid-position speed, therefore,

Minimum speed considering friction,
$\omega'_1 = \omega - 0.05\omega = 0.95\omega = 0.95 \times 47.13 = \mathbf{44.8\,rad/s}$

and maximum speed considering friction,
$\omega'_2 = \omega + 0.05\omega = 1.05\omega = 1.05 \times 47.13 = \mathbf{49.5\,rad/s}$

Mnimum radius of rotation considering friction,

$$r_1 = \mathrm{r} - \mathrm{h}_1 * \frac{\mathrm{x}}{y} = 0.0625 - \frac{0.025}{2} = \mathbf{0.0525}\boldsymbol{m}$$

$\{x = y\ and h_1 = \frac{h}{2}\}$

and maximum radius of rotation considering friction,

$$r_2 = \mathrm{r} + \mathrm{h}_2 * \frac{\mathrm{x}}{y} = 0.0625 + \frac{0.025}{2} = \mathbf{0.0775}\boldsymbol{m}$$

$\{x = y\ and h_2 = \frac{h}{2}\}$

Centrifugal force at the minimum speed considering friction,

$$F_{c1}{}' = m(\omega' 1)^2 r_1 = 5.2\ (44.8)^2\ 0.0525 = \mathbf{548}\ \boldsymbol{N}$$

and centrifugal force at the maximum speed considering friction,

$F_{c2}{}' = m(\omega' 1)^2 r_2 = 5.2\ (49.5)^2\ 0.0775 = \mathbf{987}\ \boldsymbol{N}$

S_1 = Spring force at minimum speed considering friction, and
S_2 = Spring force at maximum speed considering friction.

Minimum speed considering friction,

$$S_1 + (M.g - F) = 2F'_{c1} * \frac{\mathrm{x}}{y}$$

$$S_1 + (4 * 9.81 - 30) = 2 * 548 * 1$$

$\{x = y\}$

$$\boldsymbol{S_1} = \mathbf{1086.76}\ \boldsymbol{N}$$

Maximum speed considering friction,

$$S_2 + (M.g + F) = 2F'_{c2} * \frac{\mathrm{x}}{y}$$

$$S_2 + (4 * 9.81 + 30) = 2 * 987 * 1$$

$\{x = y\}$

$$\boldsymbol{S_2} = \mathbf{1904.76}\ \boldsymbol{N}$$

Stiffness of the spring,

$$s = \frac{S_2 - S_1}{h} = \frac{1904.76 - 1086.76}{25} = \mathbf{32.72\ N/mm}$$

3. Initial compression of the spring

We know that initial compression of the spring

$$= \frac{\boldsymbol{S_1}}{\boldsymbol{s}} = \frac{\mathbf{1086.76}}{\mathbf{32.72}} = \mathbf{33.2\ mm}$$

Problem6.6

In a spring loaded governor of the Hartnell type, the mass of each ball is 5 kg and the lift of the sleeve is 50 mm. The speed at which the governor begins to float is 240 rpm., and at this speed the radius of the ball path is 110 mm. The mean working speed of the governor is 20 times the range of speed when friction is neglected. If the lengths of ball and roller arm of the bell crank lever are 120 mm and 100 mm respectively. If the distance between the centre of pivot of bell crank lever and axis of governor spindle is 140 mm, determine the initial compression of the spring taking into account the obliquity of arms. If friction is equivalent to a force of 30 N at the sleeve, find the total alteration in speed before the sleeve begins to move from mid-position.

$\boldsymbol{Solution.}\, Given: \; m = 5\, kg$;
$h = 50\, mm = 0.05\, m$;
$N1 = 240\, rpm.\, or \omega 1 = 2\pi \times \frac{240}{60} = \mathbf{25.14} \frac{\boldsymbol{rad}}{\boldsymbol{s}}$;
$r1 = 110\, mm = 0.11\, m$;
$x = 120\, mm = 0.12\, m$;
$y = 100\, mm = 0.1\, m$;
$r = 140\, mm = 0.14\, m$;
$F = 30\, N$

Initial compression of the spring taking into account the obliquity of arms

First of all, let us find out the maximum speed of rotation ($\omega 2$) in rad/s. We know that mean working speed,

$$\omega = \frac{\omega_1 + \omega_2}{2}$$

and range of speed, neglecting friction

$$= \omega_2 - \omega_1$$

Since the mean working speed is 20 times the range of speed, therefore

$\omega = 20\,(\omega_2 - \omega_1)$

$\frac{\omega_1 + \omega_2}{2} = 20\,(\omega_2 - \omega_1)$

$25.14 + \omega_2 = 40(\omega_2 - 25.14)$

$\omega_2 = 1030.74 \; or \; \mathbf{26.43 \; rad/s}$

The minimum and maximum position of the governor balls is shown in Fig. 6p.5 (*a*) and (*b*) respectively.

r_2 = Maximum radius of rotation.
Lift of the sleeve,

$$h = (r_2 - r_1)\frac{y}{x}$$

$$r_2 = r_1 + h * \frac{x}{y} = 0.11 + 0.05 * \frac{0.12}{0.1} = \mathbf{0.17 \; m}$$

We know that centrifugal force at the minimum speed,

$$FC1 \; = \; m\,(\omega1)^2\, r1 \; = \; 5\,(25.14)^2 0.11 \; = \; \mathbf{347.6 \; N}$$

and centrifugal force at the maximum speed,

$$FC2 \; = \; m\,(\omega2)^2\, r2 \; = \; 5\,(26.43)^2\, 0.17 \; = \; \mathbf{593.8 \; N}$$

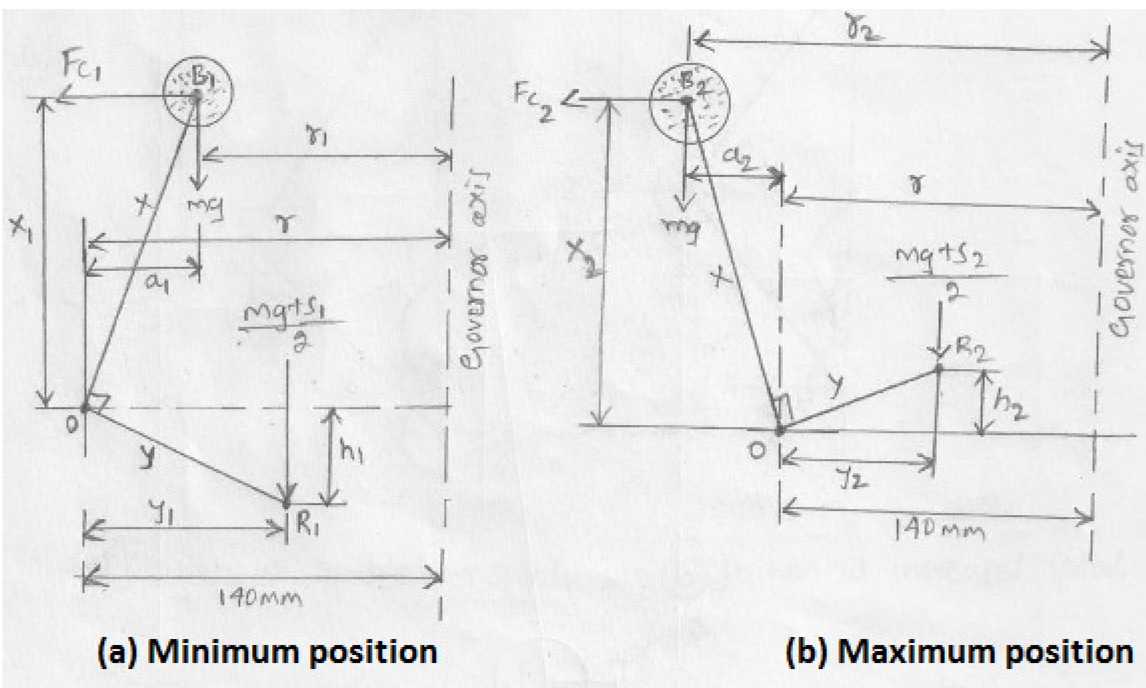


Fig 6p.5

Since the obliquity of arms is to be taken into account, therefore from the minimum position as shown in Fig. 6p.5 (*a*),

$$a1 \; = \; r - r1 \; = \; 0.14 - 0.11 \; = \; \mathbf{0.03 \; m}$$

$$x_1 = \sqrt{x^2 - (a_1)^2 =} \sqrt{0.12^2 - (0.03)^2} = \mathbf{0.1162\ m}$$

$$y_1 = \sqrt{y^2 - (h_1)^2 =} \sqrt{0.1^2 - (0.025)^2} = \mathbf{0.0986\ m}$$

$$\{h_1 = \frac{h}{2} = 0.025m\}$$

Similarly, for the maximum position, as shown in Fig. 6p.5 (*b*),

$$a2 = r2 - r = 0.17 - 0.14 = \mathbf{0.03\ m}$$
$$\therefore\ x2 = x1 = 0.1162\ m \ldots (a2 = a1)$$
$$and y2 = y1 = 0.0986\ m \ldots (h2 = h1)$$

Now taking moments about point O for the minimum position as shown in Fig. 6p.5 (a),

$$\frac{M.g + s_1}{2} * y_1 = F_{c1} * x_1 - m.g * a_1$$

$\{M = 0\}$

$$\frac{S_1}{2} * 0.0968 = 347.6 * 0.1162 - 5 * 9.81 * 0.03 = 38.9\ N$$
$$\boldsymbol{S_1 = 804\ N}$$

Similarly, taking moments about point O for the maximum position as shown in Fig. 6p.5(b),

$$\frac{M.g + S_2}{2} * y_2 = F_{c2} * x_2 - m.g * a_2$$

$\{M = 0\}$

$$\frac{S_2}{2} * 0.0968 = 593.8 * 0.1162 + 5 * 9.81 * 0.03 = 70.47\ N$$
$$\boldsymbol{S_2 = 1456\ N}$$

We know that stiffness of the spring

$$s = \frac{S_2 - S_1}{h} = \frac{1456 - 804}{50} = \mathbf{13.04\ N/mm}$$

3. Initial compression of the spring

We know that initial compression of the spring

$$= \frac{S_1}{s} = \frac{804}{13.04} = \mathbf{61.66\ mm}$$

Total alternation in speed when friction is taken into account

spring force for the mid-position,

$$S = S1 + h1.s = 8.4 + 25 \times 13.04 = \mathbf{1130\ N} \ldots..$$

$$\left\{\left(h1 = \frac{h}{2} = 25\ mm\right)\right\}$$

and mean angular speed

$$\omega = \frac{\omega_1 + \omega_2}{2} = \frac{25.14 + 26.43}{2} = \mathbf{25.785\ rad/s}$$

$$N = \frac{\omega * 60}{2\pi} = \frac{25.785 * 60}{2 * \pi} = \mathbf{246.2\ rpm}$$

Speed when the sleeve begins to move downwards from the mid-position,

$$N' = N\sqrt{\frac{S - F}{S}} = 246.2\sqrt{\frac{1130 - 30}{1130}} = \mathbf{243\ rpm}$$

and speed when the sleeve begins to move upwards from the mid-position,

$$N'' = N\sqrt{\frac{S + F}{S}} = 246.2\sqrt{\frac{1130 + 30}{1130}} = \mathbf{249\ rpm}$$

Alteration in speed

$$= N'' - N' = 249 - 243 = \mathbf{6\ rpm}.$$

6.5.Controlling Force

It is the resultant of all the external forces which oppose the centrifugal force. It can be regarded as single radial inward force acting on the centre of ball. When the ball is in equilibrium the controlling force is equal, in magnitude, to the centrifugal force acting on the ball.

When a body rotates in a circular path, there is an inward radial force or centripetal force acting on it. In case of a governor running at a steady speed, the inward force acting on the rotating balls is known as ***controlling force.*** It is equal and opposite to the centrifugal reaction. Controlling force, $FC = m.\omega^2.r$

The controlling force is provided by the weight of the sleeve and balls as in Porter governor and by the spring and weight as in Hartnell governor (or spring controlled governor). When the graph between the controlling force (*F*C) as ordinate and radius of rotation of the balls (r) as abscissa is drawn, then the graph obtained is known as ***controlling force diagram.*** This diagram enables the stability and sensitiveness of the governor to be examined and also shows clearly the effect of friction.

Controlling Force Diagram for Porter Governor

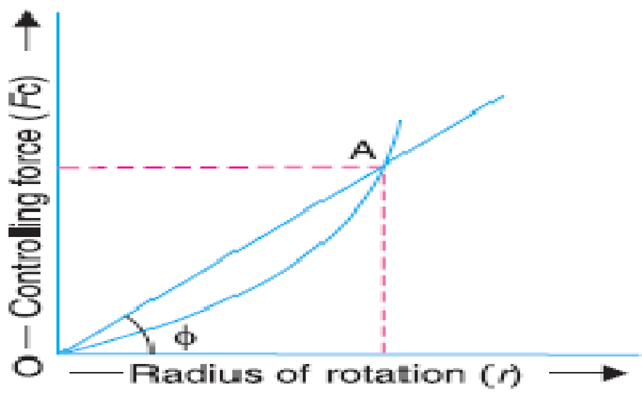


Fig 6.6

The controlling force diagram for a Porter governor is a curve as shown in Fig. 6.6. We know that controlling force,

$$F_C = m.\omega^2.r = m.\left(\frac{2\pi N}{60}\right)^2.r$$

Or

$$N^2 = \frac{1}{m}\left(\frac{60}{2\pi}\right)^2\left(\frac{F_C}{r}\right) = \frac{1}{m}\left(\frac{60}{2\pi}\right)^2(tan\phi)$$

$$\left\{\frac{F_C}{r}\right\} = tan\phi$$

$$\boldsymbol{N = \frac{60}{2\pi}.\left(\frac{tan\phi}{m}\right)^{1/2}} \quad \ldots\ldots\ldots..(1)$$

Where Φis the angle between the axis of radius of rotation and a line joining a given point (say A) on the curve to the origin O.

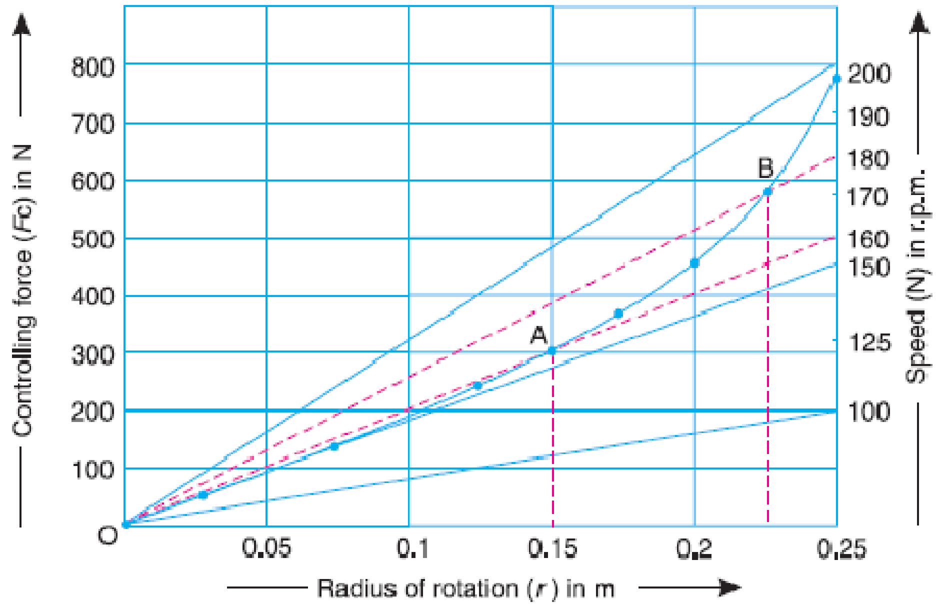
Notes : 1. In case the governor satisfies the condition for stability, the angle φ must increase with radius of rotation of the governor balls. In other words, the equilibrium speed must increase with the increase of radius of rotation of the governor balls.

2. For the governor to be more sensitive, the change in the value ofφover the change of radius of rotation should be as small as possible.

3. For the isochronous governor, the controlling force curve is a straight line passing through the origin. The angle φwill be constant for all values of the radius of rotation of the governor. From equation ***(1)***

$$tan\phi = \frac{F_c}{r} = \frac{m.\omega^2.r}{r} = m.\omega^2 = m\left(\frac{2\pi N}{60}\right)^2 = C.N^2$$

$$C = m\left(\frac{2\pi}{60}\right)^2 = constant$$



Using the above relation, the angle ϕ may be determined for different values of *N* and the lines are drawn from the origin These lines enable the equilibrium speed corresponding to a given radius of rotation to be determined. Alternatively, the same results may be obtained more simply by setting-off a speed scale along any arbitrarily chosen ordinate. The controlling force is calculated for one constant radius of rotation and for different arbitrarily chosen values of speed. The values thus obtained are set-off along the ordinate that corresponds to the chosen radius and marked with the appropriate speeds.

Controlling Force Diagram for Spring-controlled Governors

The controlling force diagram for the spring controlled governors is a straight line, as shown in Fig. 6.6a. We know that controlling force,

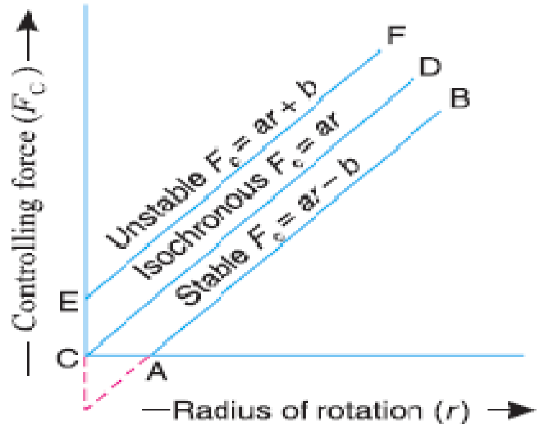


Fig 6.6a

$$F_C = m.\omega^2.r$$

6.6. Characteristics of Governors

Different governors can be compared on the basis of following characteristics.

6.6.1 Stability

A governor is said to be stable when for every speed within the working range there is a definite configuration i.e. there is only one radius of rotation of the governor balls at which the governor is in equilibrium. For a stable governor, if the equilibrium speed increases, the radius of governor balls must also increase.

6.6.2 Sensitiveness of Governors

If a governor operates between the speed limits N1 and N2, then sensitiveness is defined as the ratio of the mean speed to the difference between the maximum and minimum speeds. Thus,

N1 = Minimum equilibrium speed,
N2 = Maximum equilibrium speed, and

N = Mean equilibrium speed= $\frac{N_1+N_2}{2}$
Sensitiveness of the governor

$$= \frac{N_2 - N_1}{N} = \frac{2(N_2 - N_1)}{N_1 + N_2}$$

$$= \frac{2(\omega_2 - \omega_1)}{\omega_1 + \omega_2}$$

6.6.3. Isochronous Governors

A governor is said to be ***isochronous*** when the equilibrium speed is constant (*i.e*. range of speed is zero) for all radii of rotation of the balls within the working range, neglecting friction. The isochronism is the stage of infinite sensitivity.

Let us consider the case of a Porter governor running at speeds N_1 and N_2 rpm.

$$N_1{}^2 = \frac{m + \frac{M}{2}(1+q)}{m} * \left(\frac{895}{h_1}\right) \dots\dots..(1)$$

$$N_2{}^2 = \frac{m + \frac{M}{2}(1+q)}{m} * \left(\frac{895}{h_2}\right) \dots\dots\dots(2)$$

For isochronism, range of speed should be zero *i.e*. $N_2 - N_1 = 0$ or $N_2 = N_1$. Therefore from equations **(*1*)** and (***2*)**, $h_1 = h_2$, which is impossible in case of a Porter governor. Hence a ***Porter governor cannot be isochronous.***

Now consider the case of a Hartnell governor running at speeds N_1 and N_2 r.p.m.

$$M.g + s_1 = 2F_{c1} * \frac{x}{y} = 2 * m\left(\frac{2\pi N_1}{60}\right)^2 r_1 * \frac{x}{y} \dots\dots\dots.(3)$$

$$M.g + s_2 = 2F_{c2} * \frac{x}{y} = 2 * m\left(\frac{2\pi N_2}{60}\right)^2 r_2 * \frac{x}{y} \dots\dots\dots.(4)$$

For isochronism, N2 = N1. Therefore from equations (3) and (4),

$$\frac{M.g + s_1}{M.g + s_2} = \frac{r_1}{r_2}$$

Note : The isochronous governor is not of practical use because the sleeve will move to one of its extreme positions immediately the speed deviates from the isochronous speed.

6.6.5 Hunting

Hunting is the name given to a condition in which the speed of the engine controlled by the governor fluctuates continuously above and below the mean speed. It is caused by a governor which is too sensitive and which, therefore, changes by large amount the supply of fuel to the engine.

The following points, for the stability of spring-controlled governors, may be noted

1. For the governor to be stable, the controlling force (F_C) must increase as the radius of rotation (*r*) increases,

i.e. F_C / r must increase as *r* increases. Hence the controlling force line *AB* when produced must intersect the controlling force axis below the origin, as shown in Fig. 6.6a. The relation between the controlling force (F_C) and the radius of rotation (*r*) for the ***stability*** of spring controlled governors is given by the following equation

$$F_c = a.r - b \ldots..(1)$$

where a and b are constants

2. The value of *b* in equation (*i*) may be made either zero or positive by increasing the initial tension of the spring. If *b* is zero, the controlling force line *CD* passes through the origin and the governor becomes ***isochronous*** because F_C/r will remain constant for all radii of rotation.

The relation between the controlling force and the radius of rotation, for an ***isochronous governor*** is, therefore,

$$F_c = a.r$$

3. If *b* is greater than zero or positive, then F_C/r decreases as *r* increases, so that the equilibrium speed of the governor decreases with an increase of the radius of rotation of balls, which is impracticable Such a governor is said to be ***unstable*** and the relation between the controlling force and the radius of rotation is, therefore

$$F_c = a.r + b$$

6.5.5 Effort and Power of a Governor

The effort of a governor is the mean force exerted at the sleeve for a given percentage change of speed* (or lift of the sleeve). It may be noted that when the governor is running steadily, there is no force at the sleeve. But, when the speed changes, there is a resistance at the sleeve which opposes its motion. It is assumed that this resistance which is equal to the effort, varies uniformly from a

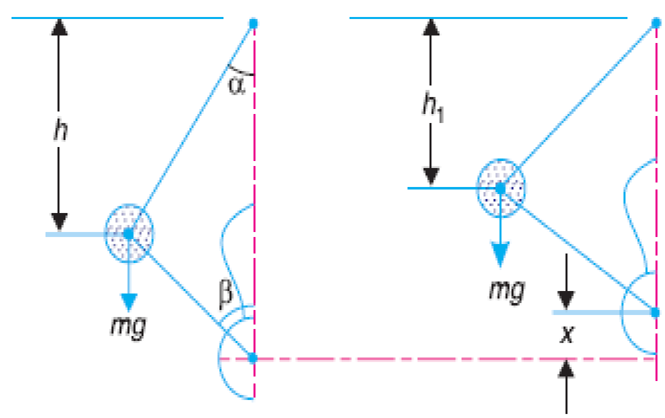
maximum value to zero while the governor moves into its new position of equilibrium.
The power of a governor is the work done at the sleeve for a given percentage change of speed. It is the product of the mean value of the effort and the distance through which the sleeve moves.

i.e., Power = Mean effort × lift of sleeve

Effort and Power of a Porter Governor

The effort and power of a Porter governor may be determined as discussed below.
Let *N* = Equilibrium speed corresponding to the configuration as shown in Fig. 6.7 (*a*), and
$c = Percentage\ increase\ in\ speed.$
$Increase\ in\ speed = c.N$
$and\ increased\ speed = N + c.N = N(1 + c)$

The equilibrium position of the governor at the increased speed is shown in Fig. 6.7 (*b*).



(*a*) Position at equilibrium speed. (*a*) Position at increased speed.

Fig 6.7

WehavediscussedthatwhenthespeedisN rpm.,thesleeveloadisM.g. Assumingthattheanglesαandβareequal, sothatq = 1, thentheheightofthegovernor,

$$h = \frac{m + M}{m} * \frac{895}{N^2} (in\ meters) \ldots \ldots \ldots \ldots \ldots (1)$$

When the increase of speed takes place, a downward force *P* will have to be exerted on the sleeve in order to prevent the sleeve from rising. If the speed increases to (1 + c) N rpm.and the height of the governor remains the same, the load on the sleeve increases to $M_1.g$. Therefore

$$h = \frac{m + M}{m} * \frac{895}{(1+c)^2 N^2} (in\ meters) \dots \dots \dots \dots \dots (2)$$

Equating equation (1) and (2) we have

$$m + M = \frac{m + M_1}{(1+c)^2}$$

$$M_1 - M = (m + M)(1 + c)^2 - m - M = (m + M)[(1 + c)^2 - 1] \dots \dots \dots (3)$$

A little consideration will show that (M_1 – M)$g$is the downward force which must be applied in order to prevent the sleeve from rising as the speed increases. It is the same force which acts on the governor sleeve immediately after the increase of speed has taken place and before the sleeve begins to move. When the sleeve takes the new position as shown in Fig. 6.7 (*b*), this force gradually diminishes to zero.

Let
P = Mean force exerted on the sleeve during the increase in speed or the effort of the governor.

$$P = \frac{(M_1 - M)g}{2} = (m + M)[(1 + c)^2 - 1]g$$

$$= \frac{(m + M)[1 + c^2 + 2c - 1]g}{2} \dots \dots \dots \dots \dots \dots \dots (4)$$

$\{Neglecting\ c^2\ being\ very\ small\}$

If F is the frictional force (in newtons) at the sleeve, then
$P = c\ (m.g + M.g \pm F)$

We have already discussed that the power of a governor is the product of the governor effort and the lift of the sleeve.

$Letx = Liftofthesleeve.$
$Governorpower = P \times x \dots \dots \dots \dots \dots . . (5)$

If the height of the governor at speed N is h and at an increased speed (1 + c) N is h_1, then

$$x = 2(h - h_1)$$

As there is no resultant force at the sleeve in the two equilibrium positions, therefore

$$h = \frac{m+M}{m} * \frac{895}{N^2} \text{ and } h_1 = \frac{m+M}{m} * \frac{895}{(1+c)^2 N^2}$$

$$\frac{h_1}{h} = \frac{1}{(1+c^2)}$$

Or $h_1 = \frac{1}{(1+c^2)} * h$

We know that

$$x = 2(h - h_1) = 2\left[h - \frac{h}{(1+c^2)}\right] = 2h\left[1 - \frac{1}{(1+c^2)}\right]$$

$$= 2h\left[\frac{1 + c^2 + 2c - 1}{1 + c^2 + 2c}\right] = 2h\left(\frac{2c}{1+2c}\right) \ldots\ldots\ldots..(6)$$

$\{Neglecting\ c^2\ being\ very\ small\}$

Substituting the values of P and x in equation (5), we have

$$Governor\ power = c(m+M)g * 2h\left(\frac{2c}{1+2c}\right) = \frac{4c^2}{1+2c}(m+M)g.h \ldots.(7)$$

$\boldsymbol{Notes}:\ \mathbf{1}. If \alpha is not equal to \beta, i.e.$

$$tan\beta\ /\ tan\alpha = q, then the equations\ (\mathbf{1})\ and\ (\mathbf{2})\ may be written as$$

$$h = \frac{m + \frac{M}{2}(1+q)}{m} * \left(\frac{895}{N^2}\right) \ldots\ldots\ldots\ldots..(8)$$

When speed increases to (1 + c) N and height of the governor remains the same, then

$$h = \frac{m + \frac{M_1}{2}(1+q)}{m} * \left(\frac{895}{(1+c)N^2}\right) \ldots\ldots\ldots\ldots..(9)$$

From equations (8) and (9), we have

$$\frac{m + \frac{M}{2}(1+q)}{m} = \frac{m + \frac{M_1}{2}(1+q)}{(1+c)^2}$$

$$\frac{M_1}{2} = \frac{m(1+c)^2}{1+q} + \frac{M}{2}(1+c)^2 - \frac{m}{1+q}$$

$$\frac{M_1}{2} - \frac{M}{2} = \frac{m(1+c)^2}{1+q} + \frac{M}{2}(1+c)^2 - \frac{m}{1+q} - \frac{M}{2}$$

$$= \left[\frac{m}{1+q} + \frac{M}{2}\right][(1+c)^2 - 1]$$

$$Governor\ effort\ P = \left(\frac{M_1 - M}{2}\right)g = \left[\frac{m}{1+q} + \frac{M}{2}\right][1 + c^2 + 2c - 1]g$$

$$= \left(\frac{m}{1+q} + \frac{M}{2}\right)(2c)g = \left(\frac{2m}{1+q} + M\right)c.g$$

$\{Neglecting\ c^2\ being\ very\ small\}$

The equation (6) for the lift of the sleeve becomes

$$x = (1+q)h\left(\frac{2c}{1+2c}\right)$$

$$Governor\ power\ =\ P * x = \left(\frac{2m}{1+q} + M\right)c.g(1+q)h(\frac{2c}{1+2c})$$

$$= \left(\frac{2c^2}{1+2c}\right)[2m + M(1+q)]g.h$$

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